Depreciation of Public Utility Property —

DEPRECIATION OF PUBLIC UTILITY PROPERTY

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A COLLECTION OF PAPERS

BY

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PREFACE

The electric public utility industry in the United States, with an investment of about $14,000,000,000 in physical plant and property, is today facing a number of serious problems as a result of changes made during the last few years in the accounting classifications prescribed by state and federal regulatory commissions. The most serious of these problems is undoubtedly that resulting from the change from the option of utilizing retirement expense accounting to the requirement of depreciation accounting. Reserves accumulated under the old accounting classifications average about eleven per cent of the investment in physical property. Such reserves are substantial, and in some companies they are probably adequate to comply with the new accounting requirements. In the case of other companies, however, reserves are not sufficient to correspond with the "actual" depreciation existing in the property. A similar situation exists in the case of many companies in the gas utility industry.

The problem of transition from the old requirements to the new carries with it many related problems of public utility law, regulation, accounting and finance, and no solution has yet been advanced that is fully acceptable to all of the various interests affected, including the public utility companies, the state and federal regulatory and securities commissions, and the consuming and investing public.

It has seemed to the authors, therefore, that in view of the importance and present urgency of these problems, it would be of interest to students of this subject to make available at this time in a single volume several papers on the subject of public utility depreciation which have been written during the period 1934 to 1939 by men who, in the course of their professional practice, have been engaged in the study of some practical public utility depreciation problems.

Another reason has suggested itself as a justification for the compilation and presentation of these papers. During the last few years much has been heard about proposals to convert the methods of accounting of public bodies, particularly of the federal government, to a basis analogous to that used by private corporations. Some writers on this subject have suggested that if this were done the improvement budgets and expenditures should be separated from the operating budgets and expenditures. It is further argued that if all of the permanent improvements constructed during the last few years were capitalized, budget deficits either would

not appear at all or would be inconsequentially small. A common characteristic of such writings has been either a complete disregard of or an inadequate provision for the cost of depreciation, including obsolescence, on the cumulative average balance of depreciable permanent improvements. Such cost would have to be provided for, however, in the accounts of any such private corporation as these writers suggest as a model for government accounting. It is likely that much more will be heard of budget and accounting reform along these lines within the next few years and it is important that students interested in the subject be prepared to help in developing a rational policy of depreciation accounting in public finance, with a view to preventing misleading and specious presentations of the problem by political interests from whatever angle. This situation and its striking analogy to the problem now facing the public utility industry have seemed to the authors a second reason justifying the compilation and presentation of these papers at this time.

No attempt has been made to edit these papers or to give them any greater coherence and unity than they already have through their relation to a common subject and through their authorship by men who have been working together in the same field, and frequently on the same problems. Instead, it has been felt that they might possibly have additional value, particularly for a student of the trend of thought in this field, if these papers were published in the form, and more or less in the order in which they were written, even though inconsistencies, duplication and lack of unity should be found to exist in them. With this thought in mind, the authors are adding the following brief statement regarding the origin and history of the several papers.

For a great many years Maurice R. Scharff has been strongly convinced that a satisfactory solution of this troublesome problem of public utility depreciation could be found only by starting from the premise that, in valuation and rate cases, accrued depreciation and annual allowance for depreciation should be treated consistently. During 1933 and 1934 he presented testimony along this line in two rate cases in Pennsylvania and Ohio. In connection with this testimony the suggestion was advanced that such consistency might be attained by estimating the annual depreciation expense by adding together the "unrealized depreciation expense," computed by dividing the percentage of accrued depreciation found to exist in the property by its weighted average age; and the "realized depreciation expense," computed on the basis of the average percentage of retirement loss to fixed capital balance over a suitable period of years. Part I of the first paper included in this volume on "The

Interdependence of Annual and Accrued Depreciation in Regulation of Public Utility Corporations" and Appendix A were written at this time in the attempt to explain this point of view and were privately circulated to a few interested friends.

As a result of criticism of this paper and of cross-examination in these cases, followed by further study, it was realized that the suggested solution was over-simplified; and that the annual depreciation determined in accordance with it, involved a certain amount of duplication or overlapping, to the extent that the unrealized depreciation existing at the beginning of each year in the property retired during such year, was included in the computation of realized depreciation. Accordingly, Part II and Appendices B and C were added to the paper, thus formulating a suggestion as to the avoidance of such duplication. Part III was added about the same time for the purpose of discussing certain related aspects of the problem.

At about the same time, Mr. Scharff in a lecture before the evening class in Public Utility Economics in the School of Commerce of New York University, summarized these thoughts, and these are included in the second paper of this group.

During the development of these early concepts, Franklin J. Leerburger was associated with Mr. Scharff as Principal Assistant Engineer and together they devoted much thought and effort to the problem of overcoming the recognized difficulties and unsatisfactory aspects of the attempted formulations, especially with respect to their failure to distinguish between those causes of depreciation that might reasonably be assumed to progress more or less in proportion to the passage of time and those which became effective irregularly with respect to time. Much consideration was also given to possible ways of taking into account the effect on depreciation losses of variations in maintenance expenditures. In this connection Joseph Jeming, who had become associated with Mr. Scharff as Statistician, made a number of helpful suggestions particularly as to mathematical methods of making allowance for such variations of maintenance expenditures. The mathematical demonstrations which were ultimately completed by Mr. Jeming resulted in improved formulations subsequently embodied in depreciation studies carried out for several public utility companies. The third, fourth and fifth papers included in this volume were written for the purpose of explaining the revised formulations to audiences of accountants, engineers and lawyers, respectively.

During the preparation of the latter group of papers, and later, after

Mr. Leerburger had joined the staff of the Public Service Commission of New York, the attempt was made by him to restate the entire argument in economic, accounting and engineering terms for the purpose of reconciling inconsistencies and discussing some of the aspects of the formulations more carefully than had previously been done. The latter work was performed in connection with Mr. Leerburger's studies at Columbia University for the degree of Doctor of Philosophy and the resultant thesis was submitted and accepted. This paper has been included as Part Two of this volume for the purpose of presenting as a whole the entire subject touched upon from various points of view by the more or less disconnected papers previously referred to, and in addition for the purpose of developing the economic basis for the suggested approach to the problem.

There has also been included at the end of Part One of this volume a paper on "An Asymptotic Method of Determining Annual and Accrued Depreciation" by Mr. Jeming. Here a novel method of approaching straight-line depreciation, when actuarial data are lacking, is presented, and one which has been found helpful in income tax and other cases. This is followed by a discussion of straight line depreciation which was prepared by Mr. Scharff for the purpose of pointing out the relationship between this paper, with its ingenious suggestion, and the subject matter of the other papers.

Finally, it has been the hope that the reading of these papers, with their obvious imperfections, might suggest to younger men in the engineering, accounting, statistical and legal professions lines along which investigations and analyses might be carried out with a view to accomplishing some further progress toward a final solution of this troublesome problem. To these younger workers and to the success of their efforts, this volume is dedicated.

THE AUTHORS

New York, N.Y.

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PART ONE

THE INTERDEPENDENCE OF ANNUAL AND ACCRUED DEPRECIATION IN REGULATION OF PUBLIC UTILITY CORPORATIONS

By Maurice R. Scharff

THE INTERDEPENDENCE OF ANNUAL AND ACCRUED DEPRECIATION IN REGULATION OF PUBLIC UTILITY CORPORATIONS

Part I

Since the beginning of the regulation of public utility corporations in the United States and the establishment of regulatory commissions to perform this function, no subject has absorbed a larger proportion of the attention of those engaged in this field than the subject of depreciation. Legislative bodies in enacting statutes to define the powers, rights and duties of the regulatory authorities and of public service companies; commissions and courts required to interpret these statutes in their opinions; commissions and associations engaged in the development of uniform accounting systems; and students of the subject—engineers, lawyers, accountants, economists, publicists, writers and speakers almost without number—have analyzed, argued, recommended and proclaimed until the literature on the subject is so voluminous that the hardiest author might well hesitate before contributing anything further.

And yet it may be stated that despite the attention that has been paid to the subject and the continuous controversy that has raged with respect to it, especially during the past twenty years, no problem in the entire field of public utility regulation is further from solution. Nor with respect to any other subject in this field has there been less satisfactory progress toward a consensus of informed opinion and an accepted standard of interpretation and application. It is this fact, together with an earnest desire to contribute however slightly toward such progress, that is responsible for the temerity of the present writer in submitting the observations which follow.

To recall briefly a few of the high spots of the long drawn out controversy leading up to the present situation, it will be remembered that prior to and at the beginning of the twentieth century the common practice of most railroads and public utilities was to charge replacements when made directly to operating expenses and that the making of any additional charge to operating expenses to provide in advance for future retirements was repeatedly criticized and condemned by the courts. *U.S. v. Kansas Pacific Railway Company,* 99 *U.S.* 455, 459

(1878); San Diego Land and Town Company v. Jasper, 189 U.S. 439, 446 (1903).

During the first decade of the twentieth century, the subject was brought into prominence for reconsideration by an amendment to the Interstate Commerce Act of 1906, strengthening the authority of the Interstate Commerce Commission with respect to the accounting practices of the companies under its jurisdiction, as a result of which, in 1907, depreciation accounting for equipment of steam railroads was made mandatory; by the establishment of public service commissions in Wisconsin and New York; and by the initiation of uniform accounting and rate regulation by these newly established bodies. The subjects of annual depreciation and accrued depreciation in valuation came in for increasing attention during this period, culminating in the decision of the United States Supreme Court in the Knoxville case in 1909, where the court held for the first time that it was the right, as well as the duty of a utility to collect from its customers in the rates charged for service an amount in addition to the cost of current maintenance and replacements "for making good the depreciation and replacing the parts of the property when they come to the end of their life." *City of Knoxville v. Knoxville Water Company, 212 U.S., 1, 13 (1909).*

The second decade of the century saw a rapid spread of public service commission regulation over the entire country and a corresponding extension of the controversy over depreciation. The proposal to determine the physical valuation of the steam railroads was introduced into Congress and widely debated, leading to the adoption by Congress of the Railroad Valuation Law in 1913, and the initiation of this gigantic task which is still in progress. In 1911, the American Society of Civil Engineers appointed a "Special Committee to Formulate Principles and Methods for the Valuation of Railroad Property and Other Public Utilities" and this committee presented a progress report in 1914, followed by an extensive discussion by members and friends of the Society, a draft of a supplement to the progress report prepared by the Chairman of the Special Committee, and a final report submitted in 1917. *Transactions A.S.C.E., LXXXI, 1311.* These reports and the discussions thereof in the proceedings of the American Society of Civil Engineers constitute classic contributions to the literature on valuation. Reading of them today and of the numerous other papers, discussions and books published by members of the Society during the same period strikingly confirms the impression that the problem of depreciation

has been a favorite subject for disagreement and debate from the very beginning of the controversy over public utility regulation.

Then came the World War, government operation of railroads, the debates incident to the return of railroads to private ownership, and the accumulation of court and commission decisions on the constantly increasing number of rate regulation cases. In 1920 Section XV-A was added to the Interstate Commerce Act, providing for the recapture of excess earnings of steam railroads; and in the same year, Section XX of the Act was amended to provide that "The Commission shall as soon as practicable prescribe for carriers subject to this act the classes of property for which depreciation charges may properly be included under operating expenses and the percentages of depreciation which shall be charged with respect to each of such classes of property, ***." Thereupon were started those voluminous proceedings No. 14700—Depreciation Charges of Telephone Companies, and No. 15100—Depreciation Charges of Steam Railroad Companies, in which thousands of pages of testimony and hundreds of pages of argument were added to the record of the depreciation controversy. Finally, the Interstate Commerce Commission adopted its report and order in 1926. *118 I.C.C. 295*. But on application of the railroads, both proceedings were re-opened for further hearing, and a superseding report and order were adopted in 1931. *177 I.C.C. 351*. This order, prescribing straight-line depreciation accounting, was to be effective January 1, 1933, but from time to time the effective date has been postponed until at the time of this writing, for all railroad property other than equipment it is January 1, 1936. *Order of Interstate Commerce Commission February 5, 1934*.

Meanwhile, rate cases have continued to call forth pronouncements by the courts on the subject of depreciation, of which the most important, perhaps, have been those in the Indianapolis case, emphasizing that "the testimony of competent valuation engineers who examined the property and made estimates in respect of its condition is to be preferred to mere calculations based on averages and assumed probabilities," *McArdle v. Indianapolis Water Company, 272 U.S. 400, 416 (1926)*; and the Baltimore case, holding that the allowance for annual depreciation should be based on fair value or replacement cost and not on original cost. *United Railways and Electric Company of Baltimore v. West, 280 U.S. 234, 253 (1929)*.

During this same period came the investigations of the New York Legislative Commission on Revision of the Public Service Commissions

Law in 1929 and 1930; the adoption of new legislation in New York, Wisconsin and many other states; the adoption of new classifications of accounts, prescribing straight-line depreciation in New York and Wisconsin; and finally, that excellent volume entitled "Depreciation—A Review of Legal and Accounting Problems," prepared by the staff of the Public Service Commission of Wisconsin, and submitted by the Commission to the 1933 meeting of the National Association of Railroad and Utility Commissioners.

This review summarizes so well the past discussions that have occurred and the legal principles that can be considered as established that no further analysis of these discussions and principles will be included here, other than the somewhat summary historical outline that has gone before. Instead, special reference is made to the "Conclusions respecting Accrued Depreciation" and the "Conclusions respecting Annual Depreciation Allowances" contained in that report at pages 121 and 144, respectively; and to the first of the "Suggested Modifications of Present Judicial Doctrines for the Future" at page 145, which may well serve as a text for the later comments on the subject selected for this paper. This first suggested modification reads as follows:

"1. The interdependence of accrued depreciation and annual depreciation should be recognized. Determinations of accrued depreciation and the annual allowance are at best reasonable estimates which must be based on the same fundamental assumptions if fair treatment is to be accorded the public and the companies. The estimates of experts as to accrued depreciation are based on certain assumptions of diminishing value and it is clearly unreasonable to use one set of assumptions to determine fair value and another to determine the annual allowance. These matters are functions of one another so to speak and should be treated as such."

This suggestion of interdependence of accrued depreciation and annual depreciation is in no sense new. It was implied in the decision of the United States Supreme Court in the Knoxville case in 1909. It was referred to in the first progress report of the Special Committee on Valuation of the American Society of Civil Engineers when it said "the interrelation in rate making cases between annual depreciation allowance and the depreciated value of the property should never be lost sight of." *First Report of Special Committee of A.S.C.E. on Valuation. December 1, 1913, p. 50.* It was even more specifically referred to in a proposed supplement to the progress report drafted by the Chairman of

the Committee in language which was not incorporated into the final report but which reads as follows: "This preliminary discussion indicates that there are methods of providing for depreciation which require the use of value new and other methods which require the use of depreciated value and it is therefore the Committee's view that any discussion of the subject which does not recognize the interrelation between the method of providing for depreciation and the method of valuing the property must necessarily be defective." *Draft of Statement submitted by Frederick P. Stearns, Chairman, for insertion in Progress Report of Special Committee of A.S.C.E. on Valuation, dated December 1, 1913, printed but not published by the Society, p. 18.*

These comments were referred to in a number of discussions published in the proceedings, from among which it is interesting to recall the following submitted by Honorable Milo R. Maltbie, then Commissioner of the Public Service Commission for the First District of the State of New York, now Chairman of the New York Public Service Commission:

"There is one thing in the report that has not been mentioned but which seems to be of special value. The Committee has pointed out that the question of the amount upon which a company is entitled to a fair return, whether cost to reproduce new or depreciated value, is inseparably connected with the method of computing the allowance for annual depreciation. Many writers have failed to appreciate the relationship between these two and have used one method for computing accrued depreciation and another for computing annual depreciation. The additional discussion by Mr. Stearns has emphasized the point made in the original report and demonstrates conclusively that one cannot fairly adopt one method for determining accrued depreciation and another for determining annual depreciation." *Proceedings, A.S.C.E., April, 1914, p. 1126.*

This same point of view was emphasized by the Interstate Commerce Commission in its report on telephone and railroad depreciation charges to which reference has already been made, in which the Commission presented at length the argument for a "direct and intimate connection between the accrued depreciation of a property which must be considered in its valuation and the annual charges for accrued depreciation which should be included in the accounts." *177 I.C.C. 351, 397 to 408.* Here is included only the final sentence of that argument: "Our conclusion is only that the same elements which produce depre-

ciation for accounting purposes likewise produce depreciation for valuation purposes and they cannot properly be observed and taken into account in the one case and at the same time be overlooked and neglected in the other."

In *New York Telephone Company v. Prendergast, 36 Fed. 2d, 54 (1929)*, a three judge statutory court adopted the balance in the depreciation reserve of the company, resulting from the excess of its charges to operating expense for depreciation less retirements actually made, as the best evidence before it of the amount of actual depreciation in the property and saying among other things the following: "Either the property is in fact depreciated to the extent of the depreciation reserve which has been created or it is not. If it is not, the plaintiff has been allowed to take money from its rate payers under the claim of depreciation which was not there either seen or unseen and which it will never have to admit *** when it built up its reserve it claimed the reserve as if actually depreciated. It would now take an inconsistent position about depreciation without fully establishing it and it has weakened its proof of present value accordingly. The plaintiff was right about depreciation when it created its reserve and it is wrong in its position now in its claims for a lesser sum as actual depreciation in this effort to establish fair value."

In this same New York Telephone case, the New York Public Service Commission in its final opinion, based upon the decision of the Federal Court referred to above, stated the matter as follows:

"The sole justification for the making of a charge to operating expense account in accordance with this rule is the existence of an actual expense or 'loss,' to use the accounting term applicable to cost and expense in the 'profit and loss' statement, and such a cost, expense or loss as the justification for a charge to operating expense is here defined as 'the current lessening in value of tangible property from wear and tear,' etc.; if such a cost, expense or loss actually exists then the accumulation in the reserve representing the lessening in value that has accrued but has not yet been realized through retirements must necessarily be deducted. If there has not been such a lessening in value then no actual cost, expense or loss has been incurred in excess of the actual losses realized on retirements made and no charge to operating expense in excess of such actual losses can be justified." *New York Telephone Company rates, etc., P.S.C., N.Y., P.U.R. 1930 C, 325, 344 (1930)*.

To the extent of its applicability to the point under discussion the

principle laid down in the decision of the United States Supreme Court in the Baltimore case, previously referred to, viz., that annual depreciation should be based on fair value or replacement cost, rather than original cost, may be said to be fully consistent with the idea of relationship between accrued depreciation in valuation and annual depreciation in operating expense.

The latest decisions of the Federal Courts on this subject appear to recognize this relationship more and more clearly. Thus in *Lindheimer et al. v. Illinois Bell Telephone Company,* decided April 30, 1934, the Supreme Court held that the company had failed to meet the burden of proving that its annual depreciation expense represented "merely *** the consumption of capital in the service rendered" in the face of "the disparity between the actual extent of depreciation, as ascertained according to the comprehensive standards used by the company's witnesses, and the amount of the depreciation reserve." And this principle was referred to and emphasized by the District Court of the United States, District of Maryland, in *Chesapeake and Potomac Telephone Company v. West et al.,* decided May 11, 1934, where the conclusion of the Supreme Court in the Illinois Bell case (supra) is stated to have been based "largely on the striking contrast between the limited amount of accrued depreciation (only 8% as contended for by the company) and the aggregate of the accumulated depreciation reserve, which over a period of years, had increased annually at the rate of more than $2,000,000 and represented for 1931 26% of the book cost of the property; and was more than three times the aggregate of the company's estimate of actual existing depreciation."

It is of interest to note in passing that the concurring opinion of Mr. Justice Butler in the Illinois Bell Telephone case just referred to, went beyond the majority opinion to hold that charges for depreciation to create any reserve whatever in excess of the actual requirements for the equalization of expenditures for maintenance and renewals are excessive, and by implication suggested a return to the doctrine that prevailed prior to the Knoxville case (1909).

It may appear that there is some contradiction of the principle of relationship between accrued depreciation in valuation and annual depreciation in operating expense in the opinion of the United States Supreme Court in the earlier New York Telephone Company case, *Board of Commissioners v. New York Telephone Company, 271 U.S. 23 (1926)*, where the court emphasized that property paid for out of reve-

nues obtained as a result of depreciation charges was the property of the company and that the consumers acquired no interest in the depreciation reserve or in property constructed with depreciation funds; and in the statement of the court in the Clarks Ferry Bridge Company case, *Clarks Ferry Bridge Company v. Public Service Commission, 2 P.U.R. (N.S.) 225, 231 (1934)*, that "It is recognized that accrued depreciation as it may be observed and estimated at a given time and the appropriate allowance for depreciation according to good accounting practice need not be the same." Neither pronouncement, however, is fundamentally inconsistent with the principle that in a rate case the amount deducted for accrued depreciation and the amount allowed for annual depreciation should be reasonably related one to the other. And in the Illinois Bell Telephone cases, both in that referred to above, and in the preceding appeal, *282 U.S., 157, 159,* the court expressly reconciled the New York Telephone case just referred to, stating that "the recognition of the ownership of the property represented by the depreciation reserve did not justify the continuance of excessive charges to operating expenses."

A special case of the recognition of the relationship between accrued depreciation in valuation and annual depreciation in operating expense is the practically universal agreement that sinking fund annual depreciation can be used only in connection with an undepreciated rate base, and straight-line annual depreciation only in connection with a depreciated rate base.

As an example of the position of students of the subject on this point, reference may be made to the lengthy discussion, with citations, in "Valuation of Public Service Corporations" by Robert H. Whitten, 2nd edition, edited by Delos F. Wilcox, from which the following quotations are taken:

Vol. 2, 1719–28, 1874

Page 1719—"It is clear that the amount of original investment devoted to public service is undiminished by the accumulation of a reserve, unless the reserve is turned over to the investors as free capital to use as they please. But the very definition of a sinking fund assumes that the fund remains a part of the investment as its earnings, from whatever source they come, are added to it, and have to be added to it in order to make it big enough to serve the purpose for which it was set up. The answer, then, is that the fund does not represent capital returned to the investors, and, therefore, their investment is undimin-

ished. If the depreciation reserve is accumulated with reasonable accuracy in the amount required to provide for retirements, and if this plan is followed under a public mandate or with public approval, the depreciation accrued during the period since the establishment of the fund on this basis cannot equitably be deducted from original cost in establishing the base upon which rates are to be calculated. This will be so whether the fund is invested in outside securities and property, or loaned to the utility at interest for capital additions."

Page 1874—"(12) The use of the sinking fund method of setting up annual depreciation charges is inconsistent with the deduction of accrued depreciation from cost new in arriving at the rate base."

See, also, Proposed Report of Commissioner Joseph B. Eastman in the Railroad and Telephone Depreciation Cases No. 14700 and No. 15100, *118 I.C.C. 295, 355,* where, after discussing straight-line depreciation, Commissioner Eastman states:

"The sinking fund method, on the other hand, is based upon the theory that the company is entitled to a fair return upon all property maintained in condition for proper service, regardless of the fact that a part of its service life has been consumed, and that the owners should receive no reimbursement for property consumption until the unit has finally been retired. Funds reserved from earnings by the depreciation charges, are, therefore, segregated and allowed to accumulate at interest pending such ultimate reimbursement. They cannot be used indiscriminately for corporate purposes, but must be used to produce definite and reasonably assumed earnings.

"The difference in theory is well illustrated by the following passage from the brief filed in behalf of the Public Service Commission in Oregon, that Commission being one of the advocates of the sinking fund method:

" 'Manifestly, the straight line method of calculating depreciation cannot properly be used in conjunction with an undepreciated rate base, nor should the sinking fund principle be followed where a depreciated rate base is considered. *** Nor should a utility company be asked or requested to forego its legitimate return upon the full undepreciated value of its property unless its capital is fully repaid as rapidly as it is consumed.'

"Granting the premises upon which they are based, both theories are logically sound."

Also, the final report of the Interstate Commerce Commission in the above proceedings, *177 I.C.C. 351, 410,* where the Commission said:

"As we conceive of the sinking fund method, it necessarily involves the proposition that the owners are not to be reimbursed for property consumed in service until the time when consumption becomes complete and the property is actually retired. In the meantime the funds reserved from income through the depreciation charges are allowed to accumulate at compound interest pending such ultimate reimbursement. They can be invested for this purpose in interest-bearing securities or in the company's own property. A corollary of the theory seems to be that in determining the value of the property upon which the company is entitled to a fair return no deduction should be made for accruing depreciation which is not as yet complete."

Note also the testimony of Honorable Milo R. Maltbie, now Chairman of the New York Public Service Commission, before the Interstate Commerce Commission in Docket 14700, referred to above, at pages 1428 to 1440 of the record from which the following extracts are quoted:

"A. If the sinking fund method is used for determining the annual expense, you must, in order to be consistent, allow a return upon the cost of the property, the cost of all the property, that is, the cost of the property which the investors contributed and the cost of the property which was built up out of the reserve. ***"

"Q. Under the sinking fund method would the element of observed depreciation enter into it?"

"A. Not at all. Well, it would, yes; but only so far as would be necessary to determine whether the property had been well maintained. *** But if it had been maintained up to the standard of proper maintenance by repairs and renewals, then the question of observed depreciation would not need to be considered at all."

In addition, the Public Service Commission of Wisconsin in its report on "Depreciation, A Review of Legal and Accounting Problems," previously referred to, stated as follows:

"The sinking fund theory is predicated on the assumption that the utility's investment will not be reduced by accrued depreciation but that in effect such deduction will be accomplished by the impounding of the interest on the depreciation reserve. It is, of course, necessary, to deal equitably with the company, that under this method an undepreciated rate base be used in determining the allowance for return. In many cases the interest on the sinking fund will be calculated at a lower rate than the return allowed. The difference between fair return and the

sinking fund interest rate is allowed the utility in return for the risks it assumes and for managerial services in handling the sinking fund. (Page 25.)

"Regulatory authorities have generally recognized that accrued depreciation is given due consideration by the use of sinking fund accruals in that the interest compounded on the reserve balance has more or less the same effect as deducting the accrued depreciation. The theory of the sinking fund method as explained in preceding sections of this report is that enough money will be taken out of earnings each year and put into a separate fund so that if properly invested either in the company's property or securities, or securities of other companies, and increased by the accretions of interest, it will be equal to the cost of the property less salvage when that property reaches the end of its useful life. As the theory requires the addition of interest on the reserve balance, usually at a specified rate, to the amounts set aside from operating expenses, obviously accrued depreciation cannot be deducted from cost new in finding the rate base, since such a deduction will either deprive the reserve of its interest accumulations and thus impair the capital of the utility, or the interest accruals must be set aside from the stockholders' fair return and thus confiscation results." (Pages 85–6.)

It may be stated, therefore, that the Courts and Commissions, as well as students of the problem, have recognized clearly that there is some relationship between the amount to be deducted for accrued depreciation and the amount to be allowed for annual depreciation in operating expenses in a rate case. This recognition, however, has been more or less intuitive, has had no adequate analytical and theoretical foundation, and has never been expressed in terms sufficient to permit its intelligent application to the testimony in a particular case.

It is suggested that such analytical and theoretical foundation as a guide for the practical application of this principle to the facts of a particular case may be found in a consideration of the relationships between realized and unrealized depreciation; and in the financial principle that the present value of an investment is the present worth of all the repayments of the capital sum of the investment plus all payments of income or return on the unrepaid portions of the sum of the capital investment. This is, of course, the principle upon which "investment values" of bonds, and bond tables are based.

Realized depreciation may be defined as the loss incurred as the result of retirements of fixed capital and corresponds, when measured on

the original cost basis, with the net debits in the retirement reserve of a company. When property has been retired it disappears from the physical inventory and would no longer be included in an estimate of cost of reproduction. When the provision in operating expenses is no more than sufficient to compensate for such actual losses on retirements—whether made from time to time in the years in which the retirements actually occur or whether equalized by some form of moving average over a reasonable period of years—such provision has no analogy to the repayment of any part of the capital sum of the property still in use and included in the physical inventory and estimate of cost of reproduction, and, therefore, corresponds with the allowance of a return on cost of reproduction new without any deduction for accrued depreciation.

When provision for annual depreciation is made on the basis of realized depreciation alone, the repayments are closely analogous to ordinary maintenance, the property as a whole being analogous to a unit of property, and the replacement of units being analogous to replacements of parts of units which are chargeable to maintenance in modern accounting classifications.

Unrealized depreciation may be defined as depreciation losses suffered in excess of realized depreciation, corresponding with losses in value, however measured, recognized to exist in property prior to its retirement and remaining still in service and, therefore, included in the physical inventory and the estimate of cost of reproduction. The provision in operating expenses of an amount for unrealized depreciation and in excess of amounts required for realized depreciation is analogous to the repayment of a portion of the capital value of the investment prior to its final retirement and corresponds with the earning of a return upon the unrepaid capital value measured by cost of reproduction less accrued depreciation.

Much of the confusion that has surrounded this subject in past discussions has resulted from failure to recognize that annual depreciation in operating expense includes the provision made for realized depreciation and may include, in addition, provision for unrealized depreciation. If such provision for unrealized depreciation is included, its application will result, in accounting procedure, in the accumulation of a reserve for depreciation representing the accumulated excess of the total provision made for depreciation over the amount of realized depreciation. This is true whatever may be the measure adopted for unrealized depreciation—whether observed physical deterioration, observed physical

deterioration supplemented by recognized obsoleteness and demonstrated inadequacy, straight line depreciation based on age and life, etc., etc. In every case the element of unrealized depreciation alone provides the analogy to the repayment of a portion of the capital sum invested in the property not retired and still remaining in service and the investment value of the property can equal the present worth of the repayments of portions of the capital sum, plus the present worth of return only if return is computed on the basis of cost of reproduction less accrued unrealized depreciation.

This may be illustrated by computation of the present worth of the annual depreciation and return on cost, less accrued depreciation in a case where the provision for unrealized depreciation is assumed to be made on the straight line basis over a period of ten years in the following tabulation:

TABLE I

Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—Straight Line Annual Depreciation and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Depreciated Investment Beginning of Year	(3) Annual Depreciation	(4) Accrued Depreciation End of Year	(5) 7% Return on Column (2)	(6) Return and Depreciation Columns (3) + (5)	(7) Present Worth Factor @ 7%	(8) Present Worth Columns (6) × (7)
1	$1,000.00	$ 100.00	$ 100.00	$ 70.00	$ 170.00	.9345794	$158.87849
2	900.00	100.00	200.00	63.00	163.00	.8734387	142.37050
3	800.00	100.00	300.00	56.00	156.00	.8162979	127.34247
4	700.00	100.00	400.00	49.00	149.00	.7628952	113.67138
5	600.00	100.00	500.00	42.00	142.00	.7129862	101.24404
6	500.00	100.00	600.00	35.00	135.00	.6663422	89.95619
7	400.00	100.00	700.00	28.00	128.00	.6227497	79.71196
8	300.00	100.00	800.00	21.00	121.00	.5820091	70.42310
9	200.00	100.00	900.00	14.00	114.00	.5439337	62.00844
10	100.00	100.00	1,000.00	7.00	107.00	.5083493	54.39337
		$1,000.00		$385.00	$1,385.00		$999.99994

It will be noted that the sum of the present worths of the annual depreciations and returns is exactly equal to the capital sum of the investment and that this is true only if the return is computed at the assumed rate on the amount of the original investment, less the accrued depreciation.

A similar relationship may be shown by a computation of the present

worth of the annual depreciation and return on cost new where provision is made for unrealized depreciation on the 7% sinking fund basis over a period of ten years:

TABLE II

Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—7 Per Cent Sinking Fund Annual Depreciation and 7 Per Cent Return on Investment—Equal Annual Payment Method

(1) Year	(2) Investment	(3) 7% Sinking Fund Factor	(4) Annual Depreciation	(5) 7% Return on Investment	(6) Return and Depreciation Columns (4) + (5)	(7) Present Worth Factor @ 7%	(8) Present Worth Columns (6) × (7)
1	$1,000.00	.072377	$ 72.377	$ 70.00	$ 142.377	.9345794	$133.06261
2	1,000.00	.072377	72.377	70.00	142.377	.8734387	124.35758
3	1,000.00	.072377	72.377	70.00	142.377	.8162979	116.22204
4	1,000.00	.072377	72.377	70.00	142.377	.7628952	108.61872
5	1,000.00	.072377	72.377	70.00	142.377	.7129862	101.51283
6	1,000.00	.072377	72.377	70.00	142.377	.6663422	94.87180
7	1,000.00	.072377	72.377	70.00	142.377	.6227497	88.66523
8	1,000.00	.072377	72.377	70.00	142.377	.5820091	82.86470
9	1,000.00	.072377	72.377	70.00	142.377	.5439337	77.44364
10	1,000.00	.072377	72.377	70.00	142.377	.5083493	72.37724
			$723.770	$700.00	$1,423.770		$999.99639

It will be noted that the sum of the present worths of annual depreciation and return is identically equal to the capital sum of the original investment and that this is true only if the return is figured on the full amount of the capital sum without any reduction for unrealized depreciation. This corresponds with the recognized practice of Commissions, Courts and students of the subject of using sinking fund depreciation only in conjunction with the return on cost new and, as has often been pointed out, results from the fact that the theory of the sinking fund assumes that moneys provided for this purpose will be set aside and retained for the sole purpose of the sinking fund until the time of its maturity, becoming available only at the end of the period as part of the general funds of the corporation. In this respect the calculation is similar to that which results from computing the present worth of annual depreciation and return on an investment of $1,000.00 on which no provision whatever is made for unrealized depreciation until the tenth year or just prior to the time of assumed retirement.

TABLE III

Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—All Depreciation Taken During the Last Year and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Depreciated Investment Beginning of Year	(3) Annual Depre- ciation	(4) 7% Re- turn on Column (2)	(5) Return and Depreciation Columns (3) + (4)	(6) Present Worth Factor @ 7%	(7) Present Worth Columns (5) × (6)
1	$1,000.00	$.......	$ 70.00	$ 70.00	.9345794	$ 65.42056
2	1,000.00	....	70.00	70.00	.8734387	61.14071
3	1,000.00	....	70.00	70.00	.8162979	57.14085
4	1,000.00	....	70.00	70.00	.7628952	53.40266
5	1,000.00	....	70.00	70.00	.7129862	49.90903
6	1,000.00	....	70.00	70.00	.6663422	46.64395
7	1,000.00	....	70.00	70.00	.6227497	43.59248
8	1,000.00	....	70.00	70.00	.5820091	40.74064
9	1,000.00	....	70.00	70.00	.5439337	38.07536
10	1,000.00	1,000.00	70.00	1,070.00	.5083493	543.93375
		$1,000.00	$700.00	$1,700.00		$999.99999

It will be noted again that the present worth of the sum of annual depreciations and returns is exactly equal to the capital sum of the investment where unrealized depreciation is provided for wholly in the year of retirement, corresponding with provision for realized depreciation only, and that this requires the computation of returns on the investment without any deduction for unrealized depreciation.

These calculations also throw light upon the result of the application of the sinking fund method when the rate of return and the rate of interest assumed on the sinking fund are not identical. In such case, it has been customary to consider the difference between the rate of fair return and the sinking fund interest rate as "allowed the utility in return for the risks it assumed and for managerial services in handling the sinking fund" (Public Service Commission of Wisconsin, "Depreciation, A Review of Legal and Accounting Problems," page 23). This excess, however, is exactly equal to the excess of the rate of return over the sinking fund rate applied to the accrued sinking fund depreciation. An adjustment for the present worth of these excesses, as indicated in the tabulation below exactly offsets it and brings the sum of the adjusted present worths to equal the capital sum of the investment.

TABLE IV

Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—4 Per Cent Sinking Fund Annual Depreciation, and 7 Per Cent Return on Investment with Adjustment for Excess of 3 Per Cent on Accrued Depreciation

(1) Year	(2) Investment	(3) 7% Return on Investment	(4) 4% Sinking Fund Annual Depreciation	(5) Total Columns (3) + (4)	(6) Present Worth Factor @ 7%	(7) Present Worth	(8) 3% on Annual Depreciation Compounded at 4%	(9) Present Worth Factor @ 7%	(10) Present Worth	(11) Total Columns (7) — (10)
1	$1,000.00	$ 70.00	$ 83.291	$ 153.291	.9345794	$ 143.26261	$	.9345794	$	$ 143.26261
2	1,000.00	70.00	83.291	153.291	.8734387	133.89029	2.49873	.8734387	2.18249	131.70780
3	1,000.00	70.00	83.291	153.291	.8162979	125.13112	5.09741	.8162979	4.16100	120.97012
4	1,000.00	70.00	83.291	153.291	.7628952	116.94497	7.80004	.7628952	5.95061	110.99436
5	1,000.00	70.00	83.291	153.291	.7129862	109.29437	10.61077	.7129862	7.56533	101.72904
6	1,000.00	70.00	83.291	153.291	.6663422	102.14426	13.53393	.6663422	9.01823	93.12603
7	1,000.00	70.00	83.291	153.291	.6227497	95.46192	16.57401	.6227497	10.32146	85.14046
8	1,000.00	70.00	83.291	153.291	.5820091	89.21676	19.73570	.5820091	11.48635	77.73041
9	1,000.00	70.00	83.291	153.291	.5439337	83.38014	23.02385	.5439337	12.52344	70.85670
10	1,000.00	70.00	83.291	152.291	.5083493	77.92537	26.44353	.5083493	13.44255	64.48282
		$700.00	$832.910	$1,532.910		$1,076.65181			$76.65146	$1,000.00035

In other words, it is not necessary to maintain the fiction that the excess of fair return from the sinking fund rate is compensation for the management of the sinking fund. Instead, the rule may be stated more accurately as requiring, where the rate of return and sinking fund rate differ, that sinking fund depreciation shall be used in conjunction with fair return on cost less sinking fund depreciation plus the sinking fund rate on accrued sinking fund depreciation. The calculation of present worth on this basis is shown in the following tabulation:

TABLE V

Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—4 Per Cent Sinking Fund Annual Depreciation, 4 Per Cent on Accrued Depreciation and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Investment	(3) 4% Sinking Fund Depreciation	(4) Accrued Factor 4% Sinking Fund	(5) Accrued Depreciation End of Year	(6) Depreciated Investment Beginning of Year	(7) 7% Return on Column (6)	(8) 4% on Accrued Depreciation Beginning of Year	(9) Total Columns (3) + (7) + (8)	(10) Present Worth Factor @ 7%	(11) Present Worth
1	$1,000.00	$ 83.291	1.000	$ 83.291	$1,000.000	$ 70.000	$	$ 153.291	.9345794	$ 143.26261
2	1,000.00	83.291	2.040	169.914	916.709	64.1696	3.332	150.7926	.8734387	131.70809
3	1,000.00	83.291	3.1216	260.002	830.086	58.106	6.797	148.194	.8162979	120.97045
4	1,000.00	83.291	4.2465	353.693	739.999	51.7999	10.400	145.4909	.7628952	110.9943
5	1,000.00	83.291	5.4163	451.132	646.308	45.2415	14.148	142.6805	.7129862	101.72922
6	1,000.00	83.291	6.6330	552.469	548.869	38.4208	18.045	139.7568	.6663422	93.12585
7	1,000.00	83.291	7.8983	657.859	447.532	31.3272	22.099	136.7172	.6227497	85.14059
8	1,000.00	83.291	9.2142	767.464	342.144	23.9499	26.314	133.5549	.5820091	77.73n16
9	1,000.00	83.291	10.5828	881.453	232.538	16.2776	30.698	130.2666	.5439337	70.85639
10	1.000.00	83.291	12.0061	1,000.001	118.548	8.2983	35.258	126.8473	.5083493	64.48273
		$832.910				$407.5908	$167.091	$1,407.5918		$1,000.00039

Where sinking fund depreciation is utilized, based on the remaining life and not on total life, it is necessary to compute return on cost less accrued depreciation in order to maintain the present worth of annual depreciation and return equal to the capital sum of the investment. This is illustrated by the following calculation in which the depreciated investment at the beginning of the year, column 2, is taken from column 6 of the preceding tabulation:

TABLE VI

Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—4 Per Cent Sinking Fund Annual Depreciation Applied on the Basis of Remaining Life and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Depreciated Investment Beginning of Year Table V Column (6)	(3) Remaining Life from Beginning of Year	(4) Annual Factor 4% Sinking Fund	(5) Annual Depreciation Columns (2) × (4)	(6) 7% Return on Column (2)	(7) Total Columns (5) + (6)	(8) Present Worth Factor @ 7%	(9) Present Worth
1	$1,000.000	10	.083291	$ 83.291	$ 70.000	$ 153.291	.9345794	$ 143.26261
2	916.709	9	.094493	86.623	64.1696	150.7926	.8734387	131.70809
3	830.086	8	.108528	90.088	58.106	148.194	.8162979	120.97045
4	739.998	7	.126610	93.691	51.7999	145.4909	.7628952	110.99430
5	646.307	6	.150762	97.437	45.2415	142.6785	.7129862	101.72780
6	548.869	5	.184627	101.338	38.4208	139.7588	.6663422	93.12718
7	447.532	4	.235490	105.389	31.3272	136.7162	.6227497	85.13997
8	342.142	3	.320349	109.606	23.9499	133.5559	.5820091	77.73074
9	232.537	2	.490196	113.990	16.2776	130.2676	.5439337	70.85693
10	118.548	1	1.000000	118.548	8.2983	126.8463	.5083493	64.48222
				$1,000.001	$407.5908	$1,407.5918		$1,000.00029

It may also be of interest to note that, if annual unrealized depreciation is assumed to occur irregularly, as might be the case if depreciation were measured by observed deterioration based on inspection from time to time during the life of a property, the present worth of the annual depreciation and return on cost less accrued depreciation will still be equivalent to the capital sum of the investment as indicated in the following tabulation:

TABLE VII

Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—Variable Annual Depreciation and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Investment	(3) Annual Depreciation	(4) Accrued Depreciation	(5) Depreciated Investment Beginning of Year	(6) 7% Return on Column (5)	(7) Return and Depreciation Columns (3) + (6)	(8) Present Worth Factor	(9) Present Worth Columns (7) × (8)
1	$1.000.00	$.......	$.......	$1,000.00	$ 70.00	$ 70.00	.9345794	$ 65.42056
2	1,000.00	100.00	100.00	1,000.00	70.00	170.00	.8734387	148.48458
3	1,000.00	100.00	200.00	900.00	63.00	163.00	.8162979	133.05656
4	1,000.00	....	200.00	800.00	56.00	56.00	.7628952	42.72213
5	1,000.00	....	200.00	800.00	56.00	56.00	.7129862	39.92723
6	1,000.00	....	200.00	800.00	56.00	56.00	.6663422	37.31516
7	1,000.00	400.00	600.00	800.00	56.00	456.00	.6227497	283.97386
8	1,000.00	....	600.00	400.00	28.00	28.00	.5820091	16.29625
9	1,000.00	100.00	700.00	400.00	28.00	128.00	.5439337	69.62351
10	1,000.00	300.00	1,000.00	300.00	21.00	321.00	.5083493	163.18012
		$1,000.00			$504.00	$1,504.00		$999.99996

As a matter of interest a mathematical demonstration of the theorem that the present worth of the repayments of portions of the capital sum plus the return on the unrepaid portion of the capital sum on the straight line, sinking fund, and remaining life methods as illustrated above, is equal to the capital sum, is attached hereto as Appendix A.

Consideration of these illustrations and demonstrations justifies the conclusion that the necessity of relating the annual depreciation in operating expense, and the accrued depreciation in valuation in a rate case rests upon more than intuition and a sense of fairness to the company and to the consumer; that fair value is the sum of the present worths of the amounts allowed for accruing unrealized depreciation and for return on cost less accrued unrealized depreciation; that annual depreciation must provide for realized depreciation and for accruing unrealized depreciation, measured in the same manner as accrued unrealized depreciation deducted from valuation; and that therefore

1. Retirement expense based on realized depreciation only should be used in connection with cost of reproduction new;

2. Straight line annual depreciation should be used in connection with cost of reproduction less straight line accrued depreciation;

3. Sinking fund annual depreciation, based on whole life, should be used in conjunction with cost of reproduction new where rate of return and sinking fund rate are the same. Where rate of return and sinking fund rate differ, sinking fund annual depreciation should be used in conjunction with rate of return on cost less accrued sinking fund depreciation plus sinking fund rate on accrued sinking fund depreciation;

4. Sinking fund annual depreciation, based on remaining life, should be used in conjunction with cost less accrued sinking fund depreciation;

5. Annual depreciation should be directly related to the amount of observed depreciation, however measured, when used in conjunction with cost less observed depreciation.

Part II

The principles formulated in the first part of this discussion are, in theory at any rate, of simple application in connection with retirement expense and straight line and sinking fund depreciation. Retirement expense, as has been pointed out, provides for equalizing realized depreciation only, without any provision for unrealized depreciation accruing in property prior to its retirement, and is properly used in connection with return on cost of reproduction new, without any deduction for accrued depreciation.

Straight line and sinking fund depreciation, of course, involve theoretical or hypothetical depreciation, which has been condemned by the Courts as constituting no actual loss of value; and in addition, apart from this legal objection, they require speculation, hypothesis or assumption for the determination of "useful lives," as it is clearly impossible to predict *actual* useful lives on any factual basis. Nevertheless, for the purpose of this discussion, the application of the principles developed above to these methods of computation is simple, as they involve the assumption that accrued unrealized depreciation shall equal the realized depreciation or the loss on retirement at the time of retirement; so that once the useful life is assumed to be known, the relations between annual and accrued depreciation that have been described and illustrated necessarily follow.

The problem is more complicated, however, when an intermediate

conception of depreciation, such as observed depreciation, is adopted, recognizing a partial loss of value, or accrual of unrealized depreciation in property, due to physical deterioration, recognized obsoleteness, demonstrated inadequacy and/or requirement of public authority, prior to retirement; plus an additional loss due to realized depreciation at the time of retirement for any of those causes. Such a conception would appear to correspond more closely than either retirement expense, straight line or sinking fund depreciation with that "actual depreciation" which the Supreme Court apparently had in mind when it said in the Indianapolis case, "the testimony of competent engineers who examined the property and made estimates in respect of its condition is to be preferred to mere calculations based on averages and assumed probabilities." *McArdle v. Indianapolis Water Company, 272 U.S. 400, 416 (1926)*. It is important, therefore, to consider how the principles developed herein may be applied to a particular set of facts as to observed depreciation and retirement losses.

The total unrealized, observed depreciation existing in property at any time is the sum of the accruals of such unrealized, observed depreciation during the past life of such property. And although it is not necessarily true that such accrual is proportional to the elapsed time, it may at least be reasonably assumed that such rate of current accrual per year is equal to the average rate per year at which such depreciation has accrued in the past; and such average rate is equal to the percentage of observed depreciation divided by the weighted average age of the property. (See Appendix B for demonstration of this relation.)

Retirements are ordinarily made before observed depreciation has reached one hundred per cent, or at the time when recognized obsoleteness, demonstrated inadequacy or the requirement of public authority necessitates it, so that an additional provision for realized depreciation must be made, in connection with provision for unrealized depreciation on the observed depreciation basis. The loss due to realized depreciation is the current moving average retirement loss, or the estimated retirement loss based on the current improvement program, if the moving average does not correspond with current requirements. In either case, however, such retirement loss duplicates the loss due to unrealized depreciation to the extent that unrealized observed depreciation exists in the property retired.

If the property retired is of the same average character as the property as a whole, such unrealized observed depreciation will equal the

product of the rate of accrual of observed depreciation for the property as a whole times the cost new of the property retired times the weighted average age of the property retired.

Adding algebraically the unrealized depreciation plus the realized depreciation minus the unrealized depreciation in the property retired, we have for the rate of annual depreciation, d, corresponding with an observed depreciation, D_o, and a retirement loss, R,

$$d = \frac{D_o}{AY} + \frac{R}{A} - y \frac{D_o}{AY} \cdot \frac{R}{A}$$

where A equals the cost new, or capital sum; Y equals the weighted average age in years of A to the beginning of the year; and y equals the weighted average age of R to the end of the year. (See Appendix C for demonstration.)

To illustrate this relationship, Tables VIII and IX have been prepared, showing the sum of the present worths of the annual depreciation and return on an assumed initial capital sum of $1,000.00, subject to an accruing observed depreciation of 5 per cent of cost new, and to retirement of $100.00 per year for ten years in Table VIII, and of 10 per cent of cost new for nine years plus retirement of the balance in the tenth year in Table IX.

It will be noted that in both cases the sum of the present worths is equal to the capital sum, only when such relationship between annual and accrued depreciation is maintained.

TABLE VIII

Present Worth of Annual Depreciation and Return on Investment over Period of Ten Years—Observed Unrealized Depreciation 5 Per Cent per Year, Retirement $100.00 per Year, and 7 Per Cent Return on Cost New Less Accrued Observed Unrealized Depreciation

Year	Cost New Beginning of Year	Observed Depreciation during Year	Retirement during Year	Cost New at End of Year	Observed Depreciation End of Year	Cost Less Observed Depreciation End of Year	Total Depreciation	7% Return on Cost Less Observed Depreciation End of Preceding Year	Total Depreciation and Return	Present Worth Factor at 7%	Present Worth
1	$1,000.00	$50.00	$ 100.00	$900.00	$ 45.00	$855.00	$ 145.00	$ 70.00	$ 215.00	.9345794	$ 200.93457
2	900.00	45.00	100.00	800.00	80.00	720.00	135.00	59.85	194.85	.8734387	170.18953
3	800.00	40.00	100.00	700.00	105.00	595.00	125.00	50.40	175.40	.8162979	143.17865
4	700.00	35.00	100.00	600.00	120.00	480.00	115.00	41.65	156.65	.7628952	119.50753
5	600.00	30.00	100.00	500.00	125.00	375.00	105.00	33.60	138.60	.7129862	98.81989
6	500.00	25.00	100.00	400.00	120.00	280.00	95.00	26.25	121.25	.6663422	80.79399
7	400.00	20.00	100.00	300.00	105.00	195.00	85.00	19.60	104.60	.6227497	65.13962
8	300.00	15.00	100.00	200.00	80.00	120.00	75.00	13.65	88.65	.5820091	51.59511
9	200.00	10.00	100.00	100.00	45.00	55.00	65.00	8.40	73.40	.5439357	39.92488
10	100.00	5.00	100.00	.....	.....	.....	55.00	3.85	58.85	.5083493	29.91636
			$1,000.00				$1,000.00	$327.25	$1,327.25		$1,000.00013

TABLE IX

Present Worth of Annual Depreciation and Return on Investment over Period of Ten Years—Observed Unrealized Depreciation 5 Per Cent per Year, Retirement 10 Per Cent per Year for Nine Years and Balance in Tenth Year, and 7 Per Cent Return on Cost New Less Accrued Observed Unrealized Depreciation

Year	Cost New Beginning of Year	Observed Depreciation during Year	Retirement during Year	Cost New at End of Year	Observed Depreciation End of Year	Cost Less Observed Depreciation End of Year	Total Depreciation	7% Return on Cost Less Observed Depreciation End of Preceding Year	Total Depreciation and Return	Present Worth Factor at 7%	Present Worth
1	$1,000.00	$50.00	$ 100.00	$900.00	$ 45.00	$855.00	$ 145.00	$ 70.00	$ 215.00	.9345794	$200.93457
2	900.00	45.00	90.00	810.00	81.00	729.00	126.00	59.85	185.85	.8734387	162.32858
3	810.00	40.50	81.00	729.00	109.35	619.65	109.35	51.03	160.38	.8162979	130.91786
4	729.00	36.45	72.90	656.10	131.22	524.78	94.77	43.38	138.15	.7628952	105.38988
5	656.10	32.81	65.61	590.49	147.62	442.87	82.01	36.73	118.74	.7129862	84.66504
6	590.49	29.52	59.05	531.44	159.43	372.01	70.86	31.00	101.86	.6663422	67.87330
7	531.44	26.57	53.14	478.29	167.40	310.89	61.12	26.04	87.16	.6227497	54.27660
8	478.30	23.91	47.83	430.47	172.19	258.28	52.61	21.76	74.37	.5820091	43.28702
9	430.47	21.52	43.05	387.42	174.34	213.08	45.20	18.08	63.28	.5439337	34.41940
10	387.42	19.37	387.42	.....	.....	.....	213.08	14.92	228.00	.5083493	115.90209
			$1,000.00				$1,000.00	$372.79	$1,372.79		$999.99434

Part III

The same method of analysis, it is believed, throws a helpful light on another problem—viz., the effect of changing the basis of depreciation during the life of a property, and the adjustment required to maintain equality between the capital sum and the sum of the present worths of the annual depreciation and return.

By way of illustration, Tables X and XI have been prepared, the first nine columns of each of which show the computation of the sum of the present worths of the annual depreciation and return on a capital sum of $1,000.00, where annual and accrued depreciation are based on retirement expense (no unrealized depreciation) for the first five years in Table X; on observed depreciation of 5 per cent per year for the first five years in Table XI; and on straight line depreciation with a useful life of ten years in the last five years in both tables.

It will be noted that in both cases a substantial deficit of present worth below the capital sum results, and it might well be contended that if straight line depreciation were enforced by public authority during the life of a company previously authorized or required to adopt retirement expense or observed depreciation, confiscation would result to the extent of such deficit.

Columns 10 to 12, inclusive, of each of the tables show the effect of adjusting the annual depreciation in each of the last five years to provide for making up the deficiency of accruals below the straight line basis during the first five years. It will be noted that in both cases, the sum of the present worths is now equal to the capital sum, suggesting that confiscation may be avoided in connection with the establishment of straight line depreciation, if required, by permitting the deficiency of prior accruals to be made up over the remainder of the life of the property, or over a reasonable period of years.

It should be pointed out that if a similar computation were made based upon straight line depreciation in the first five years, and a change to observed depreciation or retirement expense in the last five years, the result would be a corresponding excess of present worth over the capital sum, unless the annual depreciation were further decreased in the last five years to offset the over-accruals in the first five years.

TABLE X

Present Worth of Annual Depreciation and Return on Investment over Period of Ten Years—Retirement Expense (No Unrealized Depreciation) First Five Years, Straight Line Depreciation Last Five Years, 7 Per Cent Return on Cost Less Accrued Depreciation—Effect of Adjustment for Deficiency of Accruals

Year	Cost New Beginning of Year	Depreciation during Year	Accrued Depreciation End of Year	Cost Less Accrued Depreciation End of Year	7% Return on Cost Less Accrued Depreciation	Total Return and Depreciation	Present Worth Factor at 7%	Present Worth	Adjustment for Deficiency of Accrual	Total Return and Depreciation Adjusted	Present Worth
	Based on Retirement Expense (No Unrealized Depreciation) First 5 Years.										
1	$1,000.00	$	$.......	$1,000.00	$ 70.00	$ 70.00	.9345794	$ 65.42056	$	$ 70.00	$ 65.42056
2	1,000.00	.....	....	1,000.00	70.00	70.00	.8734387	61.14071	.....	70.00	61.14071
3	1,000.00	.....	....	1,000.00	70.00	70.00	.8162979	57.14085	.....	70.00	57.14085
4	1,000.00	.....	....	1,000.00	70.00	70.00	.7628952	53.40266	.....	70.00	53.40266
5	1,000.00	.....	....	1,000.00	70.00	70.00	.7129862	49.90903	.....	70.00	49.90903
	Based on Straight Line Depreciation, 10 Year Life, Last 5 Years.										
6	1,000.00	100.00	600.00	400.00	35.00	135.00	.6663422	89.95619	140.26	275.26	183.41413
7	1,000.00	100.00	700.00	300.00	28.00	128.00	.6227497	79.71196	130.44	258.44	160.94175
8	1,000.00	100.00	800.00	200.00	21.00	121.00	.5820091	70.42310	120.62	241.62	140.62471
9	1,000.00	100.00	900.00	100.00	14.00	114.00	.5439357	62.00844	110.80	224.80	122.27761
10	1,000.00	100.00	1,000.00	.....	7.00	107.00	.5083493	54.39357	100.98	207.98	105.72838
		$500.00			$455.00	$955.00		$643.50707			$1,000.00039

TABLE XI

Present Worth of Annual Depreciation and Return on Investment over Period of Ten Years—Observed Unrealized Depreciation at 5 Per Cent per Year for First Five Years, Straight Line Depreciation Last Five Years, 7 Per Cent Return on Cost Less Accrued Depreciation—Effect of Adjustment for Deficiency of Accruals

Year	Cost New Beginning of Year	Depreciation during Year	Accrued Depreciation End of Year	Cost Less Accrued Depreciation End of Year	7% Return on Cost Less Accrued Depreciation	Total Return and Depreciation	Present Worth Factor at 7%	Present Worth	Adjustment for Deficiency of Accrual	Total Return and Depreciation Adjusted	Present Worth
					Based on Observed Depreciation at 5 Per Cent per Year First 5 Years.						
1	$1,000.00	$ 50.00	$ 50.00	$950.00	$ 70.00	$ 120.00	.9345794	$112.14952	$	$120.00	$ 112.14952
2	1,000.00	50.00	100.00	900.00	66.50	116.50	.8734387	101.75560	.....	116.50	101.75560
3	1,000.00	50.00	150.00	850.00	63.00	113.00	.8162979	92.24166	.....	113.00	92.24166
4	1,000.00	50.00	200.00	800.00	59.50	109.50	.7628952	83.53702	.....	109.50	83.53702
5	1,000.00	50.00	250.00	750.00	56.00	106.00	.7129862	75.57653	.....	106.00	75.57653
					Based on Straight Line Depreciation, 10 Year Life, Last 5 Years.						
6	1,000.00	100.00	600.00	400.00	35.00	135.00	.6663422	89.95619	70.13	205.13	136.68516
7	1,000.00	100.00	700.00	300.00	28.00	128.00	.6227497	79.71196	65.22	193.22	120.32686
8	1,000.00	100.00	800.00	200.00	21.00	121.00	.5820091	70.42310	60.31	181.31	105.52392
9	1,000.00	100.00	900.00	100.00	14.00	114.00	.5439357	62.00844	55.40	169.40	92.14316
10	1,000.00	100.00	1,000.00	.....	7.00	107.00	.5083493	54.39357	50.49	157.49	80.06091
		$750.00			$420.00	$1,170.00		$821.75359			$1,000.00034

Appendix A

Mathematical Demonstration of Theorem That the Present Worth of Repayments of Portions of a Capital Sum Plus Return on Unrepaid Portion of Such Capital Sum Is Equal to the Capital Sum

Let A = A capital sum
r = Rate of return
L = Term of years over which repayment is made
n = A number of years

I. Repayment on Straight Line Basis, Return on Cost Less Repayment

Repayment plus return in

1st Year $$rA + \frac{A}{L}$$

2nd Year $$r\left(A - \frac{A}{L}\right) + \frac{A}{L}$$

3rd Year $$r\left(A - \frac{2A}{L}\right) + \frac{A}{L}$$

nth Year $$r\left[A - \frac{(n-1)A}{L}\right] + \frac{A}{L}$$

Last Year $$r\left[A - \frac{(L-1)A}{L}\right] + \frac{A}{L}$$

Present worths of above sums

1st Year $$\frac{rA + \frac{A}{L}}{1+r}$$

2nd Year $$\frac{rA + \frac{A}{L} - \frac{rA}{L}}{(1+r)^2}$$

3rd Year $$\frac{rA + \frac{A}{L} - \frac{2rA}{L}}{(1+r)^3}$$

nth Year $$\frac{rA + \frac{A}{L} - \frac{r(n-1)A}{L}}{(1+r)^n}$$

Last Year $$\frac{rA + \frac{A}{L} - \frac{r(L-1)A}{L}}{(1+r)^L}$$

Summation of present worths

$$= \sum_{n=1}^{n=L} \frac{rA + \frac{A}{L} - \frac{r(n-1)A}{L}}{(1+r)^n}$$

$$= A\left[\sum \frac{(rL + r + 1)}{L(1+r)^n} - \sum \frac{rn}{L(1+r)^n}\right]$$

$$= A\left(\left[\frac{(rL + r + 1)\,[(1+r)^L - 1]}{rL(1+r)^L}\right]\right.$$

$$\left. - \frac{r}{L}\left[\frac{1}{1+r} + \frac{2}{(1+r)^2} + \frac{3}{(1+r)^3} + \cdots \frac{L}{(1+r)^L}\right]\right)$$

To add

$$\frac{1}{1+r} + \frac{2}{(1+r)^2} + \frac{3}{(1+r)^3} + \cdots \frac{L}{(1+r)^L}$$

Multiply by $\frac{1}{1+r}$

$$\frac{1}{(1+r)^2} + \frac{2}{(1+r)^3} + \cdots \frac{L-1}{(1+r)^L} + \frac{L}{(1+r)^{L+1}}$$

Subtract

$$\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \cdots \frac{1}{(1+r)^L} - \frac{L}{(1+r)^{L+1}}$$

$$= \frac{\frac{1}{(1+r)^{L+1}} - \frac{1}{(1+r)}}{\frac{1}{(1+r)} - 1} - \frac{L}{(1+r)^{L+1}}$$

$$= \frac{(1+r) - (1+r)^{L+1}}{-r(1+r)^{L+1}} - \frac{rL}{r(1+r)^{L+1}}$$

$$= \frac{(1+r)^{L+1} - (1+r) - rL}{r(1+r)^{L+1}}$$

Summation of present worths

Divide by $1 - \frac{1}{(1+r)} = \frac{r}{(1+r)}$

$$= \frac{(1+r)^{L+1} - (1+r) - rL}{r^2(1+r)^L}$$

Then Summation of present worths

$$= A\left[\frac{(rL + r + 1)[(1 + r)^L - 1]}{rL(1 + r)^L} - \frac{(1 + r)[(1 + r)^L - 1] - rL}{rL(1 + r)^L}\right]$$

$$= A\,\frac{rL(1 + r)^L - rL + (1 + r)[(1 + r)^L - 1] - (1 + r)[(1 + r)^L - 1] + rL}{rL(1 + r)^L}$$

$$= A\,\frac{rL(1 + r)^L}{rL(1 + r)^L}$$

$$= A$$

II. Repayment on Sinking Fund Basis, Return on Cost New

Repayment plus return in each year

$$rA + \frac{rA}{(1 + r)^L - 1}$$

Present worth, nth year

$$\frac{rA + \dfrac{rA}{(1 + r)^L - 1}}{(1 + r)^n}$$

Summation of present worths

$$\sum_{n=1}^{n=L} \frac{rA + \dfrac{rA}{(1 + r)^L - 1}}{(1 + r)^n} =$$

$$\left\{A\left[r + \frac{r}{(1 + r)^L - 1}\right]\right\}\left\{\frac{1}{1 + r} + \frac{1}{(1 + r)^2} + \frac{1}{(1 + r)^3} + \cdots \frac{1}{(1 + r)^L}\right\}$$

1. $$S = \frac{1}{(1 + r)} + \frac{1}{(1 + r)^2} + \frac{1}{(1 + r)^3} + \frac{1}{(1 + r)^4} + \cdots \frac{1}{(1 + r)^L}$$

2. $$\frac{S}{1 + r} = \frac{1}{(1 + r)^2} + \frac{1}{(1 + r)^3} + \frac{1}{(1 + r)^4} + \cdots \frac{1}{(1 + r)^{L+1}}$$

Line 2 from Line 1

$$S - \frac{S}{1 + r} = \frac{1}{1 + r} - \frac{1}{(1 + r)^{L+1}} \quad \text{or}$$

$$S\left(1 - \frac{1}{1 + r}\right) = \frac{1}{1 + r}\left[1 - \frac{1}{(1 + r)^L}\right]$$

$$S\,\frac{(1 + r - 1)}{1 + r} = \frac{1}{1 + r}\left[1 - \frac{1}{(1 + r)^L}\right] \quad \text{or} \quad S = \frac{(1 + r)^L - 1}{r(1 + r)^L}$$

Subst. in first line

$$\Sigma = A\left[\frac{r(1 + r)^L - r + r}{(1 + r)^L - 1}\right]\left[\frac{(1 + r)^L - 1}{r(1 + r)^L}\right]$$

$$= A$$

III. Repayment Based on Sinking Fund over Remaining Life, Return on Cost Less Sinking Fund Repayment

Repayment to end of $(n - 1)$ years

$$= A\,\frac{r}{(1 + r)^L - 1} \times \frac{(1 + r)^{n-1} - 1}{r}$$

$$= A\,\frac{(1 + r)^{n-1} - 1}{(1 + r)^L - 1}$$

Return and Depreciation, nth Year

$$rA\,\frac{(1 + r)^L - 1 - (1 + r)^{n-1} + 1}{(1 + r)^L - 1}$$

$$+ rA\,\frac{(1 + r)^L - (1 + r)^{n-1}}{[(1 + r)^{L-n+1} - 1][(1 + r)^L - 1]}$$

$$= rA\,\frac{(1 + r)^{2L-n+1} - 2(1 + r)^L + (1 + r)^{n-1} + (1 + r)^L - (1 + r)^{n-1}}{[(1 + r)^{L-n+1} - 1][(1 + r)^L - 1]}$$

$$= rA\,\frac{(1 + r)^L[(1 + r)^{L-n+1} - 1]}{[(1 + r)^{L-n+1} - 1][(1 + r)^L - 1]}$$

$$= rA\,\frac{(1 + r)^L}{(1 + r)^L - 1}$$

Present Worth, Return and Depreciation, nth Year

$$= rA\,\frac{(1 + r)^L}{(1 + r)^n[(1 + r)^L - 1]}$$

Summation of Present Worths

$$\sum_{n=1}^{n=L} rA\,\frac{(1 + r)^L}{(1 + r)^n[(1 + r)^L - 1]}$$

$$= rA \frac{\dfrac{\dfrac{1}{(1+r)}}{(1+r)^L - 1} - \dfrac{(1+r)^{L-1}}{(1+r)^L - 1}}{\dfrac{1}{1+r} - 1}$$

$$= rA \frac{(1+r)^L - 1}{r[(1+r)^L - 1]}$$

$$= A$$

Appendix B

Mathematical Demonstration of Theorem That Average Rate of Unrealized Depreciation Is Equal to the Percentage of Observed Depreciation Divided by the Weighted Average Age

Let A = a capital sum = ΣA_n
$= A_1 + A_2 + A_3 + \ldots A_L$
L = a term of years extending from the date of A_1
n = any intermediate year between date of A_1 and L
A_n = portion of A installed in nth year
d = rate of annual depreciation
D = annual depreciation = dA
d_0 = rate of annual accrual of unrealized observed depreciation
D_0 = accrued unrealized observed depreciation
R = retirement loss during L years = ΣR_n
R_n = portion of R retired in nth year
Y = weighted average age of A
y = weighted average age of R

$$D_0 = d_0A_1(L) + d_0A_2(L-1) + d_0A_3(L-2) + \cdots d_0A_L(1)$$

$$d_0 = \frac{D_0}{A_1(L) + A_2(L-1) + A_3(L-2) + A_4(L-3) + \cdots A_L(1)}$$

$$Y = \frac{A_1L + A_2(L-1) + A_3(L-2) + A_4(L-3) + \cdots A_L(1)}{A}$$

$$d_0 = \frac{D_0}{AY} = \frac{D_0}{A} \div Y$$

Appendix C

Mathematical Demonstration of Theorem That Rate of Annual Depreciation Corresponding with Observed Depreciation Equals Ratio of Observed Depreciation Divided by Weighted Average Age Plus Ratio of Retirement Loss Divided by Investment Minus Product of These Two Ratios Times Weighted Average Age of Retirements

$$D = dA = d_0A + R - \sum d_0R_n(L - n + 1)$$

by previous demonstration $d_0A = \dfrac{D_0}{Y}$

$$\sum_{n=1}^{n=L} d_0R_n(L - n + 1) = d_0R_1(L) + d_0R_2(L - 1) + d_0R_3(L - 2) + \cdots d_0R_L$$

by definition $$y = \frac{R_1(L) + R_2(L - 1) + R_3(L - 2) + \cdots R_L}{R}$$

$$yR = \sum R_n(L - n + 1)$$

Substitute in first line

$$dA = \frac{D_0}{Y} + R - d_0yR$$

$$= \frac{D_0}{Y} + R - \frac{D_0}{AY} \cdot y \cdot R$$

$$d = \frac{D_0}{AY} + \frac{R}{A} - y \cdot \frac{D_0}{AY} \cdot \frac{R}{A}$$

Alternative demonstration

$$D = dA = d_0A + R - \sum d_0R_n(L - n)$$

by previous demonstration $d_0A = \dfrac{D_0}{Y}$

$$\sum_{n=1}^{n=L} d_0R_n(L - n) = d_0R_1(L - 1) + d_0R_2(L - 2) + d_0R_3(L - 3) + \cdots d_0R_{L-1}$$

by definition $$y = \frac{R_1(L - 1) + R_2(L - 2) + R_3(L - 3) + \cdots R_{L-1}}{R}$$

$$yR = \sum R_n(L - n)$$

Substitute in first line

$$dA = \frac{D_0}{Y} + R - d_0 y R$$

$$= \frac{D_0}{Y} + R - \frac{D_0}{AY} \cdot y \cdot R$$

$$d = \frac{D_0}{AY} + \frac{R}{A} - y \cdot \frac{D_0}{AY} \cdot \frac{R}{A}$$

PUBLIC UTILITY DEPRECIATION

PUBLIC UTILITY DEPRECIATION

Understanding that no reporters are present, that the remarks are not to be quoted, and that the speaker is not to be cross-examined on his views except in the classroom where such cross-examination will be welcome, it is proposed to discuss the subject of depreciation in relation to six propositions which may be briefly summarized as follows:

1. Depreciation, or loss in value, is essentially a legal concept;
2. Its meaning, requiring the determination of the actual extent of incidence of the causes of depreciation, is legally determined, although later modification by statute or decision is not precluded;
3. The actual incidence of the causes of depreciation can be determined by observation and by studies of obsoleteness, inadequacy, etc.;
4. Depreciation and annual depreciation in a rate case must be treated consistently with each other;
5. Depreciation in accounting should be treated as nearly consistently as possible with depreciation in rate cases and reserves should be established and maintained to reflect as nearly as possible the actual depreciation in the property;
6. Such consistency can be attained by a series of studies and judgments, which will be briefly outlined.

The timeliness of this discussion is emphasized by the recent order of the Federal Power Commission prescribing a new system of accounts for companies engaged in the interstate transmission of electricity, and which has already been adopted by a number of state Public Service Commissions, which requires separate reserves for each group of operating property accounts (production, transmission, distribution, etc.); the recent order of the New York Public Service Commission, accepted by substantially all of the companies in New York State, requiring separate depreciation reserves for each primary operating property account; and the pendency in the New York Legislature of a new bill to authorize the Public Service Commission to prescribe uniform methods of determining and accounting for depreciation; to fix the annual rates of depreciation applicable to various classes of property; and to determine the adequacy of existing reserves and prescribe the method to be

followed in restoring inadequate reserves. All of these new developments look toward the relation of depreciation in accounting to property itself, the elimination of methods of accrual at so much per KWH or per thousand cubic feet of gas, or at a percentage of gross revenue, and the establishment of new procedures designed to produce the desired results under the supervision of regulatory authority.

1. Depreciation, or Loss in Value, is Essentially a Legal Concept

Depreciation, or loss in value, like value itself, is an attribute of the property right rather than of property itself. As an abstraction, that is with reference to those qualities of property which render it desirable for the use and enjoyment of man, value and loss in value have a factual basis. As concrete economic concepts, however measurable in quantitative units of value, they are not natural phenomena controlled by any natural law. The property right is "a relation not between an owner and a thing but between the owner and other individuals in reference to things." It arises out of the social contract; and its attributes, such as value and loss in value, are social concepts made by man to serve his purposes, the meaning of which is determined by the experience of society, for the interpretation of which we must look to law as embodied in constitutions, statutes and the opinions of competent judicial and quasi-judicial bodies. It is in this sense that it is suggested that the selection of the meaning of depreciation or loss in value from among the various meanings that are advocated is essentially a legal problem and that depreciation is essentially a legal concept.

All of the meanings or theories of depreciation that have been proposed postulate the completion of the loss in value attributed to property at the time of its retirement from use but they differ with respect to the time and rate at which such loss in value occurs.

The "retirement theory," for example, assumes that there is no loss in value if property is kept in a condition for efficient service until actual retirement and that the entire loss in value occurs at the time of actual retirement.

The "cost to restore theory" assumes a partial loss in value during use to the extent of cost to restore to a so-called new condition at any time, with the complete or partial restoration of such loss of value each time the property is restored to a so-called new condition by maintenance or replacement; resulting in partial loss immediately prior to retirement only to the extent of the then cost to restore to a so-called new condi-

tion; and the complete loss of all remaining value at the time of retirement.

The "estimated actual depreciation theory" (often referred to erroneously as the "observed depreciation theory") assumes a loss in value to the extent that physical deterioration actually occurs and to the extent that obsoleteness, inadequacy, change in use, and the requirement of public authority actually take place from time to time; with possible partial restoration of value by maintenance or replacement; and with complete loss of remaining value occurring at the time of retirement.

The "sinking fund theory" assumes a progressive loss of value with the passage of time within the period of useful life in accordance with a compound interest curve corresponding with the accumulation of a fund by a fixed annual payment and compound interest at an assumed sinking fund interest rate.

The "straight line theory" assumes a progressive loss proportional to the passage of time in the period of useful life, with no intermediate restoration of value and cumulation to complete loss by the time of retirement.

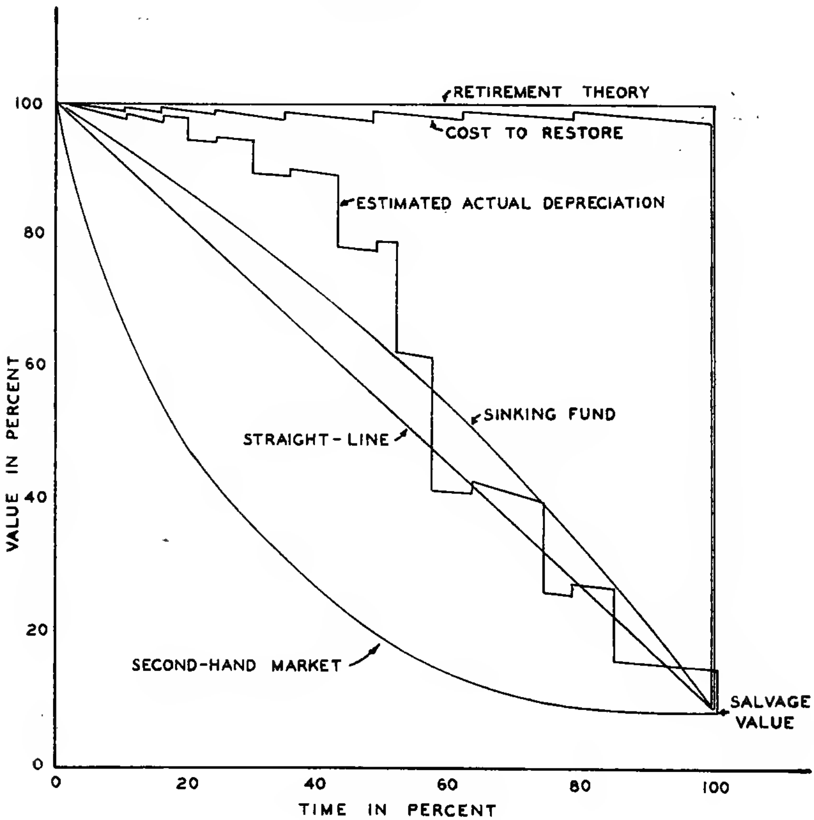
The "second-hand market theory" is sometimes proposed for particular classes of property for which the second-hand market values can be determined, such as automobiles, and while I have never known it to be advanced as a general theory of depreciation it could be so developed by assuming that immediately after the beginning of use, value declines to second-hand market value (for many classes of property this would mean scrap value less cost of removal) and continues to vary with second-hand market value until time of retirement from service.

These several theories can be illustrated diagrammatically, without any attempt to make the curves correspond with experience with any particular kinds of property, by the curves shown on the attached Plate A. Variations on these theories have been advanced and other and different theories might conceivably be proposed. The point is that substantial differences in the nature and limitations of property rights in utility property result from the application of one or the other of these interpretations of the term "depreciation." No one of them can claim any sanction in natural law. The selection of the meaning which best accords with our social experience is essentially a legal problem and it is to the law that we must look to ascertain the meaning which should be given to the term at this time and in this stage of our social development.

2. Its Meaning, Requiring the Determination of the Actual Extent of Incidence of the Causes of Depreciation, is Legally Determined, Although Later Modification by Statute or Decision is Not Precluded

Prior to the decision of the United States Supreme Court in the Knoxville case in 1909, the weight of authority seems to have supported the retirement theory and the prevailing interpretation of social experience appears to have been that the company had neither the right nor the duty to make any provision for depreciation in excess of the actual expenditures for maintenance and for the replacement, when required, of property actually retired. *U.S. v. Kansas Pacific Railway Company, 99 U.S. 455, 459 (1878)*; *San Diego Land and Town Company v. Jasper, 189 U.S. 439, 446 (1903)*.

PLATE A



In the Knoxville case in 1909 the United States Supreme Court held for the first time that it was the right as well as the duty of a utility to collect from its customers in the rates charged for services an amount in addition to the cost of current maintenance and replacements "for making good the depreciation and replacing the parts of the property when they come to the end of their life." *Knoxville v. Knoxville Water Company, 212 U.S. 1, 13 (1909).*

The long controversy over the meaning to be given to the term "depreciation" in the application of the Knoxville opinion to public utility regulation culminated in the opinion of the Supreme Court in the Indianapolis Water Company in 1926 holding that "the testimony of competent valuation engineers who examined the property and made estimates in respect to its condition is to be preferred to mere calculations based on averages and assumed probabilities." It seems to me that this opinion, supplemented and clarified by the opinion of the Supreme Court in the Illinois Bell Telephone Company case, has settled the legal principle that depreciation must be measured by determination of the incidence and extent of application of the causes of depreciation and not by the proportionate passage of any assumed period of time. This interpretation is, of course, one that will be disputed. It seems to me, however, that it accords not only with the language of the courts but also with the common experience that the incidence of the causes of depreciation may be, and often is, wholly unrelated to the passage of time. It is common knowledge that an older item of property, if well maintained, may be more valuable than an identical, younger item not so well maintained or affected by accident or other change. Furthermore, all statistical calculations of mere probabilities tend to establish relationships for all property of a particular kind rather than for the property rights in the particular property under consideration which it is the tendency of our constitution and laws to recognize.

As already indicated, however, this view that the legal principle is now established that depreciation must be measured by the incidence and extent of application of the causes of depreciation is advanced only as applying to the present time and the present stage of our social development. Substantial change in the meaning of the term seems to me to have taken place already in the past 40 years and this view does not preclude further change in the future in the nature of the property rights affected and the meaning of depreciation through the normal processes of constitutional amendment, statutory enactment and judicial interpretation.

3. The Actual Incidence of the Causes of Depreciation Can be Determined by Observation and by Studies of Obsoleteness, Inadequacy, Etc.

The causes of depreciation are generally recognized as including "wear and tear, decay, action of the elements, obsoleteness, changes in the art, inadequacy, changes in demand, and requirements of public authority," to quote the definition included in the uniform system of accounts for electric corporations prescribed by the New York Public Service Commission to be effective January 1, 1934. In my own studies it has seemed helpful to me to group these causes under five headings and to emphasize the actual incidence of each cause as a prerequisite to loss in value by describing them as actual physical deterioration; recognized obsoleteness; demonstrated inadequacy; established change in use; and prescribed requirement of public authority. In considering the provision to be made for annual depreciation perhaps a sixth cause should be added as the actual retirement of property from use to include the loss in value which occurs at time of retirement and which has not previously occurred as a result of any of the five causes of unrealized depreciation.

The total loss due to retirement of property can, of course, be readily determined at the time of retirement by adding together the book cost of the property retired and the cost of removal, and subtracting the salvage recovered. Of the unrealized losses which occur during the continuation and use of property, that caused by prescribed requirement of public authority is rarely encountered as it is ordinarily compulsory for a company to comply with such a requirement immediately when it is prescribed. Depreciation due to recognized obsoleteness, demonstrated inadequacy and established change in use can ordinarily be measured by economic studies and calculations of the savings in expense that might be made by the substitution of new or improved equipment and by consideration of the extent of impairment of the quality of service or of decline from full utilization of capacity due to the continued use of facilities which have become inadequate or which are no longer fully required because of changes in demand.

Actual physical deterioration may be determined by inspection and tests although it is difficult to relate the quantitative extent of deterioration to objective measurements. A portion of the deterioration is restorable by replacement of parts or other maintenance and may be reasonably measured by estimated cost to restore. The measurement of the

remainder of the physical deterioration which cannot be restored, excepting by replacement of the entire unit, requires a subjective synthesis of partial objective observations which is usually stated as representing the judgment of the observer as to the point on a scale between 100 per cent and 0 per cent where the condition of the property in question would fall, if all possible conditions were considered and arranged in order on such a scale. This statement, of course, is not a completely satisfactory one from the point of view of the physical sciences and raises interesting philosophical and psychometric questions which have never been fully explored. Such synthetic judgments, however, conform to human experience and it has been my personal experience that the judgments of a number of competent observers of similar training and experience with respect to an identical condition ordinarily tend to group themselves approximately about the same mean or mode.

It may be that in judging condition or extent of deterioration, we are in somewhat the same situation as men were when they attempted to convey to one another their relative sensations on touching similar objects containing different quantities of heat before it was known that temperature was proportional to molecular velocity and that a convenient scale of measurement could be established by measuring the length of a column of mercury in a glass tube. The time may come when the processes of deterioration will be sufficiently understood so that several, or hundreds, or a thousand objective measurements may be recognized as completely describing all of the types of physical deterioration that have occurred. The synthesis of these objective measurements will still be essentially psychological, however. The possibility of making such judgments completely objective depends upon the development of psychometry in directions about which at present very little is yet known.

4. Depreciation and Annual Depreciation in a Rate Case Must be Treated Consistently with Each Other

This principle has been clearly recognized by many students of the subject even from the earliest days of the controversy following the Knoxville decision. An interesting early statement on this subject was contained in a communication of Honorable Milo R. Maltbie, then Commissioner of the First District Commission of New York, to the American Society of Civil Engineers, in the course of discussion of the report of the Society's Committee on Valuation early in 1914, reading as follows:

There is one thing in the report that has not been mentioned but which seems to be of special value. The Committee has pointed out that the question of the amount upon which a company is entitled to a fair return, whether cost to reproduce new or depreciated value, is inseparably connected with the method of computing the allowance for annual depreciation. Many writers have failed to appreciate the relationship between these two and have used one method for computing accrued depreciation and another for computing annual depreciation. The additional discussion by Mr. Stearns has emphasized the point made in the original report and demonstrates conclusively that one cannot fairly adopt one method for determining accrued depreciation and another for determining annual depreciation.

Other interesting statements to this same effect were included in the report of the Committee of the American Society of Civil Engineers on valuation presented in 1913; the reports prepared by Commissioner Eastman, of the Interstate Commerce Commission on the telephone and railroad depreciation cases; the opinion of the New York Public Service Commission in the New York Telephone case, decided in 1930; the report on depreciation prepared by the staff of the Wisconsin Public Service Commission, and submitted to the 1933 meeting of the National Association of Railroad and Utility Commissioners; etc., etc.

These statements seem to me to have rested largely on intuition and a sense of fairness to the investor and to the consumer. It can be demonstrated mathematically, however (the demonstration is too long for presentation in this paper), that fair value at any time is equal to the sum of the present worths of the annual depreciation, plus return on cost less unrealized depreciation only when annual depreciation and unrealized depreciation are computed on the same basis. The correspondence of annual and accrued depreciation in a rate case, therefore, has a basis in financial mathematics which corresponds with that of bond tables, which show bond values as the sum of the present worths of interest plus repayments of principal, and which supports the soundness of the principle that depreciation and annual depreciation in a rate case must be treated consistently with one another.

5. Depreciation in Accounting Should be Treated as Nearly Consistently as Possible with Depreciation in Rate Cases and Reserves Should be Established and Maintained to Reflect as Nearly as Possible the Actual Depreciation in the Property

Since accounts are intended to reflect the actual financial results of operations during each period and the actual financial situation of the

enterprise at all times, this statement appears almost axiomatic. As the New York Public Service Commission said in the New York Telephone case, decided in 1930:

> The sole justification for the making of a charge to operating expense account in accordance with this rule is the existence of an actual expense or "loss," to use the accounting term applicable to cost and expense in the "profit and loss" statement, and such a cost, expense or loss as the justification for a charge to operating expense is here defined as "the current lessening in value of tangible property from wear and tear," etc.; if such a cost, expense or loss actually exists then the accumulation in the reserve representing the lessening in value that has accrued but has not yet been realized through retirements must be necessarily deducted. If there has not been such a lessening in value then no actual cost, expense or loss has been incurred in excess of the actual losses realized on retirements made and no charge to operating expense in excess of such actual loss can be justified.

Certainly a charge to operating expense which is in excess of the requirement to create and maintain a reserve proportionate to the depreciation in the property is greater than the loss that is actually incurred; and an accrual which is less than this requirement is improvident in that a loss or consumption of capital has taken place without absorption in expense.

The New York Public Service Commission seemed to me to call attention to this relation again in its memorandum on unification plans issued under date of November 10, 1936, when it stated that "the time and expense required to investigate any company or system could be greatly reduced if continuing property records were kept up to date, if the books of account showed the original cost, *if the depreciation reserves represented even approximately the amount of depreciation actually existing in the property* and if it were not necessary to estimate reproduction cost." (Italics supplied.)

The Committee on Utility Accounting of the American Institute of Accountants also called attention to this idea in its letter to the Federal Power Commission, dated April 1, 1936, in which they oppose the introduction of compulsory straight line depreciation in the then proposed system of accounts, as follows:

> Provision for depreciation, to have any meaning, can be made only out of earnings and, these being subject to regulation, it follows that corporate earnings cannot equitably be charged with depreciation on a basis totally unrelated to that used in determining the permitted rates which produce

such earnings. Failure to recognize this principle will, in our opinion, lead to serious and harmful confusion in corporate accounts and finance and we can only conclude that the introduction of compulsory straight-line depreciation should not be undertaken in advance of general acceptance of the method for rate regulation purposes, not only by state commissions but by the courts of final appeal.

6. Such Consistency Can be Attained by a Series of Studies and Judgments, Which Will be Briefly Outlined

It can be stated as a general rule that annual depreciation consistent with any assumed meaning or theory of depreciation will be equal to the retirement loss or realized depreciation during the year; plus the net increase in unrealized depreciation during the year in property remaining in service at the end of the year; minus the amount of unrealized depreciation existing in the property retired during the year at the time of its retirement.

This can be stated in mathematical language without adding to or detracting from its meaning as follows:

Let $\$D$ = Annual depreciation in dollars
$\$D_0$ = Unrealized depreciation in dollars
$\$A$ = Capital sum in dollars
$\$R$ = Retirements during year in dollars
$\$D_r$ = Unrealized depreciation in retired property at time of retirement in dollars
d = Total annual rate of depreciation in per cent
d_0 = Annual rate of increase of unrealized depreciation in per cent
Y = Weighted average age of $\$A$ in years

Then

$$\$D = d\$A = \$R + d_0\$A - \$D_r$$

Dividing by $\$A$, we have

$$d = \frac{\$R}{\$A} + d_0 - \frac{\$D_r}{\$A}$$

A useful study for the purpose of determining the rate of annual depreciation consistent with any actual depreciation, $\$D_0$, is to determine the average experience over a reasonable period of years prior to the date of study. Without going into the mathematical background of the formula, such average experience may be expressed as follows:

$$d_{\text{ave.}} = \frac{\Sigma \$R}{\Sigma \$A} + \frac{\$D_o}{\$AY} - \frac{\$D_o}{\$A} \times \frac{\Sigma \$R}{\Sigma \$A}$$

It is of interest to note that these may be considered general equations, all three terms of which have values if $\$D_o$ is determined by estimating the actual extent of incidence of the causes of depreciation; whereas the last two terms cancel each other on the retirement expense theory, yielding an annual provision equal to retirement loss alone; and the first and third terms cancel each other on the straight line theory, yielding an annual rate based on assumed life alone.

It is, of course, inconsistent with the idea of reserve accounting to have an exact correspondence in each year between the provision for depreciation and the depreciation loss that actually occurs in the same year. In this sense, the opinion of the United States Supreme Court in the *Clarks Ferry Bridge Company case, 2 P.U.R., N.S. 225, 231 (1934)*, is consistent with the views here expressed when it stated: "It is recognized that accrued depreciation as it may be observed and estimated at a given time and the appropriate allowance for depreciation according to good accounting practice, need not be the same." Reserve accounting contemplates correspondence of depreciation and actual depreciation over reasonable periods of years. To this end studies of average past experience which will yield rates that would have been required over the past to produce reserves corresponding with the presently determined depreciation must be supplemented by study of the budgets for the ensuing year, particularly with respect to retirements; the trend of change of the condition of the property as affected by the budget for maintenance expenditures; the extent to which existing depreciation will be restored by maintenance expenditures; the relation between the existing reserve balances and the existing depreciation; and the general aim of bringing about correspondence within a reasonable period of time between the average balances in reserve and the average actual depreciation.

As an example, if a property is found to be in 85 per cent condition with a weighted average age of 10 years, the average rate which would have been required over its past history to provide a reserve equal to the existing depreciation would have been 15 ÷ 10 or 1.5 per cent. If the ratio of total past retirements to the sum of the annual balances of fixed capital is \$100,000.00 ÷ \$1,000,000.00 or 10 per cent, an additional provision of 10 ÷ 10 or 1 per cent would have been required to take care of the past average retirement loss. If the property retired, however, was in

the same condition and at the same average age as the present property, the unrealized depreciation at time of retirement would have been 15 per cent and the necessary annual rate to balance the total loss would have been decreased by 15 per cent of 1 per cent or .15 per cent. On this basis the total average annual loss in the past would have been 1.5 per cent + 1 per cent − .15 per cent = 2.35 per cent. This, of course, is not a determination of the accrual required for the ensuing year but only a study of past average experience as a guide to judgment under existing conditions. When supplemented by consideration of the budgets for additions, retirements and maintenance, and by consideration of the other requirements that have been referred to, the conclusion might be justified that the reasonable provision for the ensuing year would be a judgment rate of 2 per cent or 2.5 per cent, or other appropriate figure. The reasonableness of such judgments could be checked from time to time and at intervals of say 5 or 10 years by re-examinations of the property and redetermination of the amount of depreciation and of the correspondence between such depreciation and the balances in the reserves.

It is suggested that such a program, if conscientiously carried out by any utility company in New York State, might go far toward meeting the requirements that have been laid down by the New York Public Service Commission in its memorandum on unification plans and might avoid an indefinite continuation of the controversy of past years between supporters of conflicting meanings or theories of depreciation.

DEPRECIATION ACCOUNTING PROBLEMS

DEPRECIATION ACCOUNTING PROBLEMS

I particularly appreciate the opportunity to discuss this subject briefly before this session of the Conference of the Accounting Section of the Edison Electric Institute, sponsored by the Plant Accounting and Records Committee, for the reason that it seems to me that the time and the place are more than usually appropriate. In fact, I am encouraged to believe that if I can interest this group at this time in the suggestions which I am going to lay before you, there is good reason to hope for some constructive progress towards the solution of this complicated and vexing problem of depreciation accounting.

Now, why do I say that this time is more than usually appropriate for the discussion of this problem? My reason is that, after nearly thirty years of continuous controversy over this subject, we are clearly approaching a crisis in the discussion; and I confidently believe that we may look forward to seeing the problem settled, at least for a long time, during the two or three years immediately in the future. It is of vital importance that this imminent settlement of the problem shall be a sound and constructive one and not an unsound and destructive one.

Why do I say that a crisis is approaching in the discussion? Simply because all of the new classifications of accounts prescribed by the Federal and State authorities have been unanimous for the first time in abandoning the attempt to prescribe either straight line depreciation accounting or retirement expense accounting, and have required, to quote the language of the Federal Power Commission System of Accounts, "each utility shall record as at the end of each month the estimated amount of depreciation accrued during that month on depreciable electric plant." To me, this requirement seems to reflect a major change in accounting policy and to represent a determination to have depreciation accounting record the depreciation or loss in value that actually occurs due to the causes of depreciation. And the vitally important corollaries of this policy, which is now in full force and effect, are the harmonization of the meanings of depreciation in valuation and in accounting; and the necessity of measuring loss of value due to the causes of depreciation as part of the practical routine of accounting procedure.

Such a policy seems to have been clearly forecast by numerous discus-

sions of the relation of depreciation in operating expense to depreciation in valuation during the course of the long controversy on the meaning of depreciation, which I will not take time to quote. More specifically, however, it was recently forecast in a memorandum on unification plans issued by the New York Public Service Commission under date of November 10, 1936, when it stated that "the time and expense required to investigate any company or system could be greatly reduced if continuing property records were kept up to date, if the books of accounts showed the original cost, *if the depreciation reserves represented even approximately the amount of depreciation actually existing in the property* and if it were not necessary to estimate reproduction cost." (Italics supplied.)

Similarly, the Committee on Utility Accounting of the American Institute of Accountants stated in its letter to the Federal Power Commission, dated April 1, 1936, in which it opposed the introduction of compulsory straight line depreciation into the then proposed system of accounts as follows: "Provision for depreciation, to have any meaning, can be made only out of earnings, and, these being subject to regulation, it follows that corporate earnings cannot equitably be charged with depreciation on a basis totally unrelated to that used in determining the permitted rates which produce such earnings. Failure to recognize this principle will, in our opinion, lead to serious and harmful confusion in corporate accounts and finance and we can only conclude that the introduction of compulsory straight line depreciation should not be undertaken in advance of general acceptance of the method for rate regulation purposes, not only by state commissions but by the courts of final appeal."

Finally, it may be worth while to recall the question put to one of the witnesses for the Edison Electric Institute by one of the accountants of the Federal Power Commission, at the hearing on the Federal Power Commission Classification of Accounts in the spring of 1936, when he asked, "Suppose the classification simply provided that each company should charge to expense and credit to depreciation reserve each year, the amount or amounts which represented the actually accruing depreciation occurring in the plant, *** what would be your view as to that kind of a provision?"

The answer of the witness, by the way, was that he wouldn't know how to go about it. But this answer is no longer adequate, as we now have exactly the kind of provision then suggested, and we must know how to go about complying with it.

If I am right in believing that the problems of depreciation in valuation and depreciation in accounting are going to be definitely settled, and settled in harmony with each other, within the next two or three years, then, as I have indicated, it is of vital importance to the industry that it be settled on a sound and reasonable basis and not upon an unsound and unreasonable one. And yet I am convinced that unless the industry takes an aggressive position in favor of a constructive and progressive program, we may expect a thoroughly unsound and unreasonable solution of the problem. This is indicated by the fact that within the last ten months, the Appellate Division of the Supreme Court of New York has in two different cases, approved findings of the New York Public Service Commission that the straight line method reflected actual depreciation both in determining fair value and in establishing the allowable charge to operating expense for rate making purposes.

It is significant, too, that in one recent rate case in New York, the accounting division of the Public Service Commission has submitted a recommendation that a company be required to increase its reserves for depreciation to the amount estimated by the Commission as straight line depreciation by debiting the surplus account with the difference. (This recommendation has not yet been acted upon by the Commission.)

In these days, in litigation involving governmental bodies, the tendency of the Courts is strong—and naturally and properly so in my opinion—to accept the thesis of the government when it is met by no adequate substitute program from the other side. It is my considered opinion, therefore, that if the utility industry adheres to the claim that there is no such thing as depreciation in a well maintained property except the cost to restore the property to efficient operating condition, and that retirement accounting with no balance in reserve adequately reflects the financial condition of a corporation, we shall see straight line depreciation firmly established in the law of the land as the measure of the loss in value of utility property from the causes of depreciation. This was clearly implied by the Appellate Division of the Supreme Court of New York in one of the recent cases, to which reference has been made, when it said, "The petitioner contended that no depreciation should be deducted, simply the cost of restoring the depreciable property to a new condition. There was ample evidence before the Commission to sustain its method of determining depreciation."

Still worse, if the industry adopts a stand-pat, obstructive attitude we shall see the dispute over the effect of deficiencies in existing reserves

below straight line depreciation reserves, as estimated by the staffs of Public Service Commissions, resolved by the recognition of such deficiencies as overstatements of corporate surplus required to be corrected by adjustment of the accounts, regardless of the creation of corporate deficits or of other effect upon the financial statements of the company. I do not need to point out to this group what such an outcome would mean in the demoralization of financial programs, and in postponement of dividends until capital has been restored by surplus earnings.

Nothing could be more unsound or unreasonable than such an outcome. Straight line depreciation has been properly held in the past to be statistically incorrect, economically unsound, hypothetical, confiscatory and illegal.

In equity and good conscience, companies which have complied in the past with the prescribed requirements of regulatory commissions and the established law of the land in utilizing retirement accounting should have a reasonable opportunity over a period of years to adjust their reserves, by charges to operating expense account, to meet the requirements of the new policy and the new interpretation which is now in the course of establishment in our regulatory procedure and law.

These considerations will be swept aside, however, and such reasonable opportunity will not be given unless a constructive substitute program is aggressively urged upon the attention of the Commission and the Courts. Fortunately the first step toward the development of such a constructive substitute program has already been taken by the accounting committee of the Edison Electric Institute when, in its memorandum submitted to the Federal Power Commission under date of April 10, 1936, it recommended that the rigid application of retirement accounting, without any requirement whatever for reserves, be modified to the extent of requiring charges to expense sufficient to establish and maintain reserves within reasonable stated upper and lower limits. It is true that the suggestion made in the memorandum was that the lower limit should be "that percentage below which adequate provision would not be made for anticipated retirements beyond a limited future period or for other than minor contingencies" and that the upper limit should be "that percentage which will provide adequate accumulations to meet both current and prospective retirements for known causes, the effects of which can be foreseen with reasonable accuracy, and for unforeseen retirements and contingencies, beyond which limit further material increases would not be clearly related to retirement needs."

It is true also that the vagueness of these definitions was brought out

in the examination of the committee witnesses, and that on cross-examination one witness for the committee agreed that "the minimum limit of the reserve might have some relation to the observed physical depreciation." The abandonment of the strict application of retirement accounting, with no requirement of any balance in reserve, however, was a great step forward and seems to me to open the door for consideration of what is a reasonable measure of the adequacy of the reserves.

My own view is that the tendencies to which I have already referred are sound and reasonable, and that from the point of view of the industry, as well as from that of the regulatory commissions, it is desirable that the amount of actual depreciation or loss in value due to the causes of depreciation existing in the property be adopted as a guide to determination of reasonable balances in reserve. The actual depreciation existing in the property at any time can be determined by inspection and observation of its condition and of the extent of physical deterioration, supplemented by engineering studies of the extent of actual existence of obsoleteness due to progress in the arts, inadequacy, change in use and requirement of public authority.

Such inspections and observations must be as complete as possible and must frankly recognize every form of physical deterioration as a cause of depreciation or loss in value regardless of whether or not it affects continued efficient operation of the property.

Engineering studies of obsoleteness must frankly and fully recognize actual progress in the arts, and must develop methods of measuring the resultant loss in value by evaluating the present worth of the savings in expense, the improvement in quality of service, and the increase of safety or reliability that would result from the utilization of the most efficient available modern equipment.

Studies of inadequacy and change in use must fully recognize these conditions where they exist and must develop suitable scales for measurement of their effects. When the actual amount of depreciation existing in a property has once been determined, it may be utilized as a basis for apportionment of existing reserves into such subdivisions as may be required by the applicable classification of accounts; and as a guide to policy in building up reserves over a period of years so as to effect the transition from retirement reserve accounting to depreciation reserve accounting. The rate at which such depreciation is continuing to accrue and the probable amount of depreciation that will exist in the property at the end of the ensuing year, can also in my opinion, be determined from year to year by continuous study of the past history of the prop-

erty, of the period within which the existing depreciation has occurred, of the effect of maintenance in retarding and restoring depreciation, and of the effect of proposed additions, retirements and maintenance expenditures upon the condition of the property. Such study will recognize that some portion of depreciation, as in the case of the deterioration of certain types of property, does progress proportionately to the passage of time; whereas other causes of depreciation, which are responsible for the great majority of retirements become effective at the times when definite events occur, such as the development of new methods or equipment, a change in method of operation or in demand, etc. The annual review will permit constant adjustment toward the desired ideal of establishing and maintaining reserves which will be the same proportion of the book value of depreciable property as the loss in value due to the causes of depreciation is to the value new; and new physical inspections and complete engineering studies at intervals of five to ten years can supplement such annual reviews so as to avoid too wide a variation between the reserve ratios and the actual facts.

Such a procedure will avoid the errors of the straight line depreciation method and the retirement accounting method alike, and will lead to reserve balances which will fall somewhere in between the inadequate amounts that correspond with retirement accounting, and the excessive amounts that correspond with the straight line method.

It is my opinion that the reasonableness of such a procedure can be successfully established and that the aggressive adoption and support of such a reasonable program is the best protection that the industry or any member company can have against the permanent establishment of the legality of straight line depreciation and against the establishment of the principle that deficiencies in reserves constitute overstatements of corporate surplus.

I suggested at the beginning of this talk that not only the time but also the place was particularly appropriate for consideration of this subject. I think that I have amply demonstrated its timeliness. With respect to the place, I will only say that I do not believe that the industry can look to any other section of its organized activities for so large a contribution to the solution of this important problem as in the case of the accountants, particularly those engaged in plant accounting and record work, and the engineers engaged in cooperation with these accountants.

It is to the members of this group that the industry must look to bring home to the executives of utility companies the financial threat

involved in the present depreciation situation. They must convince their officers and directors that adequate provision for depreciation can no longer be made secondary to and dependent upon the maintenance of the existing level of net income at present rates or at rates constantly reduced to meet the demands of public officials avid for public recognition; and that the only sound policy under present conditions is the charging of adequate provisions for depreciation and insisting upon rate structures that will support such charges in addition to a reasonable return upon the valuation of the property. Even the recognition of the inadequacy of existing reserves and the determination to devote what otherwise would be available for increase of income to the building up of reserves are likely to be futile in view of the pressure of regulatory commissions to convert every possible growth in income into rate reductions.

The members of this group also must make it clear to their companies that it is no longer possible to rely on the decisions of the courts of ten or fifteen years ago, when it was held in various cases that depreciation had no application to the property of a company which was well maintained and which was rendering efficient service. It is a comfortable form of wishful thinking, perhaps, to dwell upon the statement of the United States District Court in a New York case about 1920 that "if, in fact, the capacity has remained the same, depreciation should not be a function of the rate base at all"; and that "it (straight line depreciation) has no application while the plant is kept up"; or on decisions of the Idaho Supreme Court about 1915 to the effect that it can be demonstrated that the plant is in good operating condition, and giving as good service as a new plant, then the question of depreciation may be entirely disregarded. But no realistic view of the present situation can justify the expectation that the same point of view will now prevail.

Many difficult technical problems remain to be solved before the program here advanced can be successfully carried out. Standardization of methods of measurement of the effects of the various causes of depreciation is essential and clarification at many points is desirable. I venture to hope that the Plant Accounting and Records Committee will continue to make this problem the subject of intensive study and consideration during the next few years and that through its efforts a reasonable solution may be worked out which will be acceptable to the Commissions and the Courts, and which will contribute to the permanent financial stability and prosperity of the electric utility industry.

DEPRECIATION AND OBSOLESCENCE

DEPRECIATION AND OBSOLESCENCE

Synopsis

Depreciation should be used as a generic term for the effects of physical deterioration, obsoleteness, inadequacy and change in use or in demand, and requirement of public authority. The writer treats the engineering aspects of each of these causes of depreciation, and suggests that engineering study offers the greatest hope of progress toward their better understanding.

The determination of annual depreciation expense is then discussed. It is pointed out that such determinations heretofore have customarily been based on one or the other of two erroneous assumptions—either that all the causes of depreciation became effective in proportion to the passage of time, in accordance with a straight line or sinking-fund formula; or, that none of these causes became effective with the passage of time, or prior to the retirement of the property. The writer suggests that a rational engineering approach would recognize that some causes of depreciation do progress with the passage of time, and that others do not.

A procedure for the application of this concept to the determination of annual depreciation expense is summarized briefly, and its relation to the cost of power in competitive industry, public utilities, and governmental projects is discussed.

An opportunity is afforded the Engineering Profession to clarify the subject of depreciation by discussion of its engineering aspects. The writer proposes that civil engineers invite the co-operation of engineers in all branches of the profession in carrying out such a program.

Introduction

About a quarter of a century ago a Special Committee of the Society undertook a study of the problem of valuation of public utilities, the final report† on which has achieved recognition as an important contribution to the literature of this controversial subject. Not that this report ended the controversy. On the contrary, it has raged unabated to

the present time, and has lost none of its vigor—as has been demonstrated by the renewal late in 1937 of the old debate over the relative merits of "prudent investment" and "fair value" as a basis for public utility rate-making. One chapter of the report was devoted to the subject of depreciation; and although twenty years have elapsed since its publication, one can still agree with the opening sentence of that chapter, in which the Committee states: "Perhaps there is no single subject in connection with valuation that has caused more trouble than depreciation." *Transactions, Am. Soc. C. E., Vol. LXXXI (1917), Chapter VI, p. 1448.*

On re-reading this report, in preparation for this paper of the cost of depreciation and obsolescence in energy generation, the writer has had the impression that if depreciation had been more clearly recognized at that time as the result of the actual incidence of specific causes, the Committee would have devoted more attention to consideration of the engineering facts regarding these causes and their effects, and would have placed less emphasis upon the relative merits of mathematical formulas for relating depreciation solely to the passage of time. Much of the discussion of the subject in the past, particularly that by accountants and economists, has proceeded from the assumption that depreciation was a simple function of cost, time, and the interest rate. More attention has often been paid to the shape of the curves and the forms of the equations for these functions than to the facts which they purported to explain.

It is the writer's proposal that depreciation be used as a generic term for the effects of several categories of causes. He proposes to discuss this suggestion in somewhat general terms, with reference to cost of energy generation only for purposes of illustration, conceiving this to be the most helpful contribution he can make to this, the Second Symposium on Power Costs.

The categories of causes of depreciation usually listed in discussions of the subject include deterioration, wear and tear, obsoleteness, inadequacy, change in demand, change in use, and requirement of public authority. The mere enumeration of these causes, however, suggests that their effects cannot be expressed as any simple function of the passage of time, because it is common knowledge that an older unit of property or equipment may be, and often is, less deteriorated, more efficient, safer or more reliable, and less affected by change in demand, change in use, or the requirement of public authority than a similar unit of property or equipment of lesser age.

It is true that the incidence of these causes are events that occur in the universal framework of time, and that the effects of some of them are continuous, and sufficiently regular to permit formulation as simple functions of time. In most cases, however, the incidence of these causes is discontinuous, irregular, and even fortuitous with respect to time; and the engineer recognizes that their effects can be better expressed in terms of functions of physical and chemical change, abrasion, rupture, change of form or dimensions, loadings, stresses and moments, capacities, demands and uses, efficiencies, lost time accidents, service interruptions, competition of substitute services, etc., than in terms of the complicated, high-order functions that would be required to express their relations to the passage of time.

It is most fitting, therefore, that the Engineering Profession should devote attention to the development of a better understanding of the causes of depreciation and the measurement of their effects. If progress can be made in this direction, it may yet come about that accounting for costs and profits in industry and finance, and regulatory practices relating to public utilities will be brought into harmony with the facts relating to depreciation. It has too often been assumed that the meaning of depreciation is dependent upon accounting and regulatory policies and practices.

Part of the confusion of the past has been due to the fact that in accounting and finance, depreciation has been looked upon as synonymous with amortization of investment. It is a sound financial principle that the value of a capital investment is equal to the present worth of the amortization of the investment plus the return upon the unamortized portion of the investment. This principle can be complied with, however, by the consistent application of any method of amortization whatever, regardless of its relation or lack of relation to depreciation.

Confusion has also been caused by the controversies and contradictions that have grown up around the meaning of "fair value" in regulation of public utilities, and the legal measure of depreciation in determining "fair value." The Courts have performed a valuable service in emphasizing the reasonableness of the correspondence of such legal measure with the facts:

"The testimony of competent valuation engineers who examined the property and made estimates of its condition is to be preferred to mere calculations based on averages and assumed probabilities." *McArdle v. Indianapolis Water Co., 272 U.S. 400 (1926).*

Further evidence is given in the following:

"In the light of the evidence as to the expenditures for current maintenance and the proved condition of the property—in the face of the disparity between the actual extent of depreciation, as ascertained according to the comprehensive standards used by the company's witnesses, and the amount of the depreciation reserve—it cannot be said that the reserve merely represents the consumption of capital in the service rendered." *Lindheimer v. Illinois Bell Telephone Co., 292 U.S. 151 (1934).*

It is proposed now to discuss briefly each of the principal recognized causes of depreciation that have been enumerated as follows: (1) Physical deterioration; (2) obsoleteness; (3) inadequacy and change in use or in demand; and (4) requirement of public authority.

Physical Deterioration

Where property is used up and disappears progressively with use as, for example, the lead in a pencil or the material in a grindstone, such use is analogous to the consumption of such supplies as fuel taken from a coal pile. Such consumption, of course, can be measured in simple units of length, volume, or weight.

When the use of property can be isolated for study, it may be possible to measure the progress of deterioration as a function of such use. Wood poles, for example, have a single use—that of sustaining overhead conductors by reaction to vertical compression, on the one hand, and supporting unbalanced span loads and lateral wind loads through resistance to bending, on the other. It is suggested that deterioration through fiber rot in such a case may reasonably be measured in terms of decline in compression and bending strength at standard fiber stresses.

The foregoing examples are cases in which simple objective measurements of the effects of deterioration appear reasonably applicable. In many types of cases, however, the relation of deterioration to objective measurements is less obvious. Thus, when a steam boiler is new it can operate under a definite pressure limitation as prescribed by the A.S.M.E. standards and boiler insurance codes. Furthermore, the tubes, shells, and headers have definite thicknesses of sound metal designed to support the anticipated stresses with reasonable factors of safety. As deterioration occurs, the thickness of sound metal is reduced. Derating of boilers by insurance inspectors supplies an additional objective measure of the extent of deterioration.

Similarly, changes in the dielectric strength of insulation and in other physical and chemical characteristics occur in the windings of generator coils as a result of use and of changes in temperature and dielectric stress. Extensive measurements of some of these characteristics have been made by manufacturers and by engineering committees in an effort to describe them.

In these and in similar cases, however, it would be impossible, in the present state of knowledge, to secure agreement upon any standard series of objective measurements that would be generally accepted as measuring the extent of deterioration. Engineering estimates of conditions reflecting the extent of deterioration in such cases, therefore, are customarily stated as representing the judgment of the engineer as to the point where the deteriorated condition of the property in question would fall on a scale between 100 per cent, representing condition new, and 0 per cent, representing a condition requiring removal, if all degrees of deterioration were arranged in order of magnitude.

Such a judgment, of course, is in a measure subjective. It has been the experience of the writer, however, that such judgments of a number of competent observers of similar training and experience with respect to an identical condition tend to be reasonably consistent and to group themselves around a mean without excessive deviations.

It may be that in judging condition or extent of deterioration, engineers are in somewhat the same situation as men were when they attempted to convey to one another their relative sensations on touching similar objects at different temperature levels before it was known that temperature was proportional to molecular velocity or that a convenient scale for the measurement of temperature could be established by measuring the length of a column of mercury in a glass tube.

The time may come when the processes of deterioration will be sufficiently understood so that several objective measurements may be recognized as completely describing all the types of physical deterioration that have occurred in an item of property and may be synthesized into a determination of condition that is related as closely to objective observations as the unused length of the lead in the pencil, the remaining stone in a grindstone, the weight of fuel in a coal pile, or the remaining fiber strength of a wood pole.

The writer believes that progress in this direction depends upon engineering study, and proposes that civil engineers invite the co-operation of engineers in other branches of the profession specializing in the design, construction, and operation of power plants in undertaking to de-

vise more complete objective standards for the measurement of the effects of deterioration of power plant structures and equipment, and of suggesting improved methods of arriving at sound engineering judgments and valid statistical inferences in the light of such objective standards as are already available.

Obsoleteness

Attention is called to the use of the word "obsoleteness," instead of "obsolescence" as included in the title of this paper, in deference to the use of the latter term in the designation of the Sub-Committee of the Power Division, of which the writer is Chairman. The use of "obsolescence," derived from the inceptive Latin verb ending in "-sco," is misleading, since such verbs describe verbal actions commencing and progressing continuously with the passage of time. "Obsoleteness," derived from a past participle describing a completed verbal action, seems more suitable for describing a condition existing at a particular time, especially when it is intended to refer to a condition resulting from a cause which is discontinuous in time and which becomes effective upon occurrence of specific events, such as the development of new inventions or other steps in the progress of the arts.

It is suggested that the decline in the economic value of existing equipment, resulting from the development of improved equipment, which can be operated at a saving, may reasonably be used as a measure of obsoleteness. Economic value is defined as the sum of the present worths of the expected returns, where such returns are equal to the total revenues less all operating expenses (exclusive of provision for depreciation or amortization).

Progress in the arts tends to affect competitive enterprises by attracting into the field lower cost producers, with resultant lowering of prices, elimination of marginal producers, and decline of the profits and economic values of remaining enterprises using obsolete facilities.

With a relatively inelastic demand, such decline of economic value may be stated as the total of the cost of the old equipment plus the sum of the present worths of its operating expenses (exclusive of depreciation or amortization) over a reasonable period of time, minus the corresponding total for the most modern available improved equipment. The ratio of decline is this difference divided by the cost of the old equipment.

Algebraically, the percentage effect of such obsoleteness may be stated as follows:

$$D_o = 100 \frac{C - [C_e + \sum_{0}^{n} W_p (O - O_e)]}{C} \qquad (1)$$

in which D_o = percentage depreciation due to obsoleteness; C = cost of existing equipment; O = annual operating expenses associated with existing equipment (exclusive of depreciation); C_e = cost of the most economical modern substitute equipment; O_e = annual operating expenses associated with the most economical modern substitute equipment (exclusive of depreciation); and $\sum_{0}^{n} W_p$ = sum of the present worths of an annual sum over a reasonable period of n years.

In some instances, improved equipment results in greater safety, greater reliability, or improved quality of product, rather than in realizable savings in operating and maintenance expense, and in such cases the measurement of obsoleteness appears less simple and has the subjective character of engineering judgments to which reference has been made in connection with estimates of some forms of physical deterioration, and the judgment is expressed as a percentage on a scale between 100 per cent and zero. In this case, also, there would seem to be opportunity for the development of objective measurement by engineering study, and for progress toward an objective basis for judgment.

Inadequacy and Change in Use or in Demand

As in the case of obsoleteness, these causes of depreciation become effective as a result of specific events, such as addition of new load, loss of load, change in market, or change in method of operation. Inadequacy is rarely encountered in well managed profitable enterprises, for the reason that in such cases facilities are promptly increased to take advantage of opportunities for profitable employment of capital. Excess capacity due to decline in market or change in methods of operation, however, is not uncommon and there is opportunity for clarification of its meaning and development of improved methods of measuring its effects. The analogy of decline in price, to the extent of savings in cost of production that could be accomplished by a producer, with no more than adequate equipment, appears reasonable as a basis of measurement

and on this basis the percentage depreciation due to change in demand or in use, as before:

$$D_u = 100 \frac{C - [C_a - \sum^{n} W_p (O - O_a)]}{C} \quad \ldots\ldots\ldots\ldots (2)$$

in which D_u = percentage depreciation due to change in use; and C_a and O_a are the cost and the annual cost of operation of equipment of no more than adequate capacity.

Requirement of Public Authority

The effects of this cause of depreciation are rarely found in property in service, because compliance with such requirements is ordinarily compulsory, and retirement and replacement take place at the effective date of such requirement. Where property is maintained in contravention of legal requirement by competent public authority, either the property right in such property has in effect been destroyed, and depreciation due to this cause should be considered complete; or a liability is imposed upon the owner of the property, the amount of which may be taken as a measure of depreciation due to this cause.

Annual Depreciation Expense

If the existing depreciation in the property is the sum of the effects of the several causes of depreciation that have been discussed herein, the annual depreciation expense should be the sum needed to make the reserve for depreciation representative of existing depreciation at the beginning and at the end of each year. If it were practicable to make a depreciation study as of January 1 of each year and determine the existing depreciation, it would be possible to compute the reserve requirements at such dates. Then, the annual depreciation expense would be the difference between the required reserves at the beginning and at the end of the year plus the debits to the reserve during the year. The debits to the reserve during the year are the net retirement losses, which are equal to the difference between the sum of the cost of the property retired plus the costs of dismantling and the salvage recovered.

Of course, a complete new depreciation study as of January 1 of each year cannot be made on account of the magnitude of the task and the time required to make the determination of existing depreciation. How-

ever, it is possible to approximate, within narrow limits, the probable change in the amount of existing depreciation during a year, and thus to approximate the amount of annual depreciation expense.

The difficulty of determining the annual depreciation expense arises from the discontinuous relation of some of the causes of depreciation to time. The result of this difficulty has been that depreciation due to all causes has generally been treated either as if all these causes became effective directly and solely in proportion to the passage of time, and as if depreciation were identical with amortization of investment by a straight line or a compound interest formula, or as if none of these causes was effective in proportion to the passage of time, and as if depreciation losses occurred only at the time of retirement of property.

It is suggested that a rational engineering approach to the problem would be to recognize that the effects of some causes of depreciation on some property do proceed in a simple functional relation to time and that others do not; and to attempt to formulate a plan for computing annual depreciation which will more nearly correspond with facts than either of the erroneous treatments to which reference has been made.

The writer does not minimize the difficulties of such an approach. He believes, however, that engineering study offers promise of progress in the direction indicated which is well worth the effort of attempting to overcome the difficulties that lie in the way. With this in mind he proposes to describe briefly a procedure which he has developed and used in studies made under his direction.

Of all the causes of depreciation, physical deterioration comes the nearest, and in a greater proportion of cases, to progressing approximately in proportion to the passage of time, whereas obsoleteness, inadequacy, change in use or in demand, and requirement of public authority occur irregularly, and often at the time of the retirement of a property. Admitting freely the inaccuracy of the assumption and pending the development of more complete knowledge of the relation between incidence of the causes of depreciation with respect to the particular property and the passage of time, it is suggested that a helpful guide to the determination of annual depreciation expense in many cases will be found in a computation based upon the assumption that deterioration does progress directly in proportion to the passage of time, and that the other causes of depreciation become effective at the time of retirement and can be provided for as retirement losses.

To make such a computation it must be recognized that the physical

condition determined by an engineer, in the manner previously described, is the result of two offsetting influences: (1) Deterioration, which acts to lower the condition of the property; and (2) the replacement of parts through maintenance, which tends to restore the condition of the property.

The physical condition of the property as found is the net result of the effect of these opposing influences during the history of the property. Every dollar spent on maintenance, however, does not improve the condition of the property. Some maintenance expenditures tend only to prevent deterioration and others affect only the location or arrangement of property. The cost of replacing parts on maintenance, also, because of its piecemeal character, may be substantially greater than the cost of the parts replaced as portions of the cost of property used in operations.

If it is assumed: (1) That a constant proportion of maintenance expenditures has been effective in restoring deterioration; (2) that maintenance expenditures have been spread evenly over the property whose condition has been determined; and (3) that parts replaced on maintenance tend to deteriorate at the same average rate as all the property whose condition has been determined, then it is suggested that the relation of these several factors may be expressed by the equation:

$$K_n = 100\,[1.00 - XY_n + \frac{b_n}{B_n}\,(I)\,(1.00 - Xy_n)] \ldots\ldots\ldots (3)$$

in which K_n = percentage condition resulting from physical deterioration; X = average annual rate of deterioration in the past; Y_n = weighted average age in years of dollars representing the property; $\frac{b_n}{B_n}$ = ratio of cumulated maintenance expenditures (b_n) to fixed capital balance, representing the property (B_n); I = ratio of maintenance expenditures effective in restoration of condition; y_n = weighted average age in years of cumulated maintenance expenditures; and n = subscript indicating year, January 1 of which is the basis of the study.

Equation (3) may be solved for X, which is the average rate of deterioration independent of the offsetting effects of maintenance. The reciprocal of X may be considered as an average indicated useful physical life, which the property would live if the constant rate of deterioration continued from 100 per cent to 0 per cent condition, and if it were retired solely because of physical deterioration.

With the same assumptions as before, and with amounts budgeted for additions, retirements, and maintenance expenditures for an ensuing

year following the date for which existing depreciation has been determined, the annual depreciation expense, D_a, may be stated as follows:

$$D_a = [B_{n+1}(1.00 - K_{n+1}M_n) - B_n(1.00 - K_nM_n)] + R_n \ldots (4)$$

in which M_n = the net percentage effect of all causes of depreciation other than deterioration; R_n = the estimated net retirement loss in the ensuing year; and K_{n+1} is expressed by Equation (3) for $n = n + 1$ (the subscript, $n + 1$, indicates merely that the figures to which they are applicable are those of a year later than n).

The writer has found this formulation useful (although it is inchoate), in estimating the costs required to be budgeted for depreciation expense during a year following a determination of existing depreciation. Where depreciation reserves have been established in accounts that are the same proportion of the book cost of depreciable property as the percentage depreciation at the beginning of the year determined by engineering study in the manner described, this procedure will tend to preserve such correspondence between reserves and actual depreciation at the end of the ensuing year.

The procedure, of course, is capable of indefinite refinement, as information and a better understanding of the facts of the particular case become available. If there is ground for anticipating that deterioration of some part of the property will not proceed in proportion to the progress of time, correction may be made on this account. If the causes of obsoleteness, inadequacy, or change in use or in demand are known to be in operation in a manner that will result in additional depreciation during the ensuing year, due to these causes, without justifying retirement, a proportionate allowance may be made for these factors. It may be desirable in any case to increase the estimated requirement for depreciation expense by a percentage for contingencies to allow for the obvious limitations on the accuracy of the procedure. If such a procedure should be followed for a number of years it would probably be desirable, possibly at intervals of five to ten years, to check the correspondence between the resultant reserves and the actual depreciation of the property by a reexamination of its condition and by new engineering studies of obsoleteness and of the effects of change in demand or in use.

It does not seem worth while, at this time, to labor the point, or to attempt to develop, in any more detail, the solution of the practical problems that will be encountered in the application of this procedure. The purpose is to emphasize the opportunity for a contribution by the Engineering Profession to a rational solution of this problem which it is be-

lieved will have application to competitive industry, to public utilities, and to the determination of the cost of power developed by public projects, to which it is now proposed to make brief reference.

Competitive Industry

In competitive industry the prices of products tend to be determined by equating utility to the marginal consumer to cost to the marginal producer. Profit is the difference between price, as so determined in the market, and cost of production. Neither price nor profit has any necessary relation to the investment in fixed facilities of the particular enterprise under consideration; and the value of the enterprise is the present worth of its prospective earnings including profit and recoupment of investment at such time and in such manner as the profitable operation of the enterprise in the competitive market may make possible.

The policy of management, however, as to amortization of investment may be based on considerations other than the occurrence of actual depreciation and retirement losses. Thus, if a uniform product is manufactured and sold, it may be desirable that the entire cost of amortization of investment—or possibly that all costs—shall be spread over the total number of units of the product, regardless of the time when, and the rates at which, such costs or losses are actually incurred. On the other hand, a more limited equalization of the effect of depreciation on cost of production in accordance with some other plan may be desirable because of some other special consideration. It has seemed to the writer that it is for this reason that depreciation and amortization of investment have been so generally considered as identical in accounting practice.

It is suggested, however, that knowledge of the actual time and rate of occurrence of losses due to depreciation, based upon competent engineering study, will be helpful to management even when amortization policies are not controlled by depreciation. The tendency to extend governmental supervision over financial and accounting practices in connection with the issue of securities, moreover (as illustrated by developments under Security and Exchange Acts), suggests the possibility that even in competitive industry a closer correspondence between such practices and the actual facts of depreciation may be necessary or desirable at some time in the future.

Public Utilities

In the public utility industry the situation seems to be quite different. The principle has been established that public utility rates are to be based

upon allowance of gross revenues from sales of service equal to operating expense, plus annual depreciation, plus a reasonable return on the fair value of the property used and useful in the public service. Fair value, as prescribed by the Courts, must be based upon consideration of reproduction cost less depreciation, prudent investment less depreciation, and other pertinent measures of value. The tendency of the Courts, in prescribing these general requirements, has been to insist that depreciation deducted in determining fair value shall be "actual depreciation," and that there must be a reasonable correspondence between such "actual depreciation" and the reserves resulting from the annual depreciation included in operating expense.

The freedom of competitive industry to amortize investment in any manner suggested by the special conditions of the industry is limited by the definite requirement that depreciation charges and reserves shall correspond as closely as possible with the facts as to depreciation losses and the condition of the property from time to time, taking into account all the effective causes of depreciation.

The tendency toward clarification of this requirement has received a marked impetus in the two years, 1936–1937, through adoption by various Federal and State regulatory commissions of new classifications of accounts requiring (to quote the language of the classification of the Federal Power Commission, for example) that "each utility shall record as at the end of each month the estimated amount of depreciation accrued during that month on depreciable electric plant."

The method of approach herein outlined complies with these special requirements which are applicable to the public utility industry. Even the suggested methods of measuring the effects of obsoleteness and excess capacity due to change in demand or in use, derived from the analogy of the economic effect of such changes on price and economic value in the competitive field, seem to be directly applicable. The regulation of semi-monopolistic public utility enterprises constitutes, in effect, the substitution of an artificial simulation of competitive conditions to make up for the lack of the influence of free competition on prices.

The references previously made to opportunities for refinement of methods of calculation, with increasing knowledge of facts and their relations, are especially applicable in the present case. Some equalization of requirements for annual depreciation expense by averaging over reasonable periods of years may also be desirable. In special cases, as where the property of a public utility consists of a single long-lived unit, like a hydro-electric dam, it may even be desirable, as a matter of public policy,

to equalize such costs by a compound interest formula over a long period of years.

However, even such departures of amortization provision from depreciation, if desirable, cannot be made intelligently, and cannot avoid that unreasonable disparity between existing depreciation and reserves which has been condemned by the Courts, unless the facts are fully understood on the basis of proper engineering investigation and analysis. Such engineering investigation and analysis may well provide a more generally accepted basis of determining amortization based upon average depreciation experience for the purpose of making economic comparisons of different types of design and of the relative costs of steam and of hydro-electric power. The importance of such development is illustrated by the widespread discussion that followed the publication in November, 1937, of the report of the Power Authority of the State of New York on "Government Hydro v. Private Steam Power." In this report it was shown that the average fixed charges of four modern base-load steam-generating stations, furnishing power to distributing companies under contract, included depreciation varying from 1.90 per cent to 3.25 per cent. The Authority adopted 2.0 per cent as a reasonable comparative allowance for depreciation and renewals on both public and private water power plants, and 3.0 per cent, based on sinking-fund accrual at 7 per cent interest, as a reasonable comparative allowance for private steam generating stations.*

Cost of Power Produced by Governmental Projects

As in the case of competitive industry, the amortization of investments in public projects of any character may be controlled by considerations of policy unrelated to the occurrence of depreciation in the property representing such investments. It is not intended to suggest that it is a function of this Society, or of any other engineering organization, as such, to discuss such governmental policies. It is suggested, however, that, as in the case of competitive industry, knowledge of the engineering facts relating to the occurrence of depreciation in such property will be helpful to all those concerned in such policies, including governmental officers, tax-payers, and voters; and it is obvious that if helpful comparisons are to be made of costs of power produced by governmental projects, and of those produced by private agencies, it will be helpful to include costs of depreciation in both on a strictly comparable basis.

* H.R. Committee on Rivers and Harbors, Doc. No. 52, 75th Cong., 2d Session, pp. 26, 37, 38.

Conclusion

The purpose of the writer in presenting this discussion of depreciation has been primarily to emphasize the engineering aspects of the problem, to which he believes sufficient consideration has not been given in the past, and to suggest that an opportunity be afforded the Engineering Profession to clarify this highly controversial subject by discussion of its engineering aspects. He ventures to express the hope that civil engineers will take the lead in this engineering activity, and will invite the cooperation of engineers in all branches of the profession in carrying forward a program which, he is convinced, can be made of great value, both to the profession and to the public.

PUBLIC UTILITY DEPRECIATION

PUBLIC UTILITY DEPRECIATION

The problem of public utility depreciation is in the forefront of every consideration of any aspect of the regulation of public utilities at the present time (May, 1938). The Federal Communications Commission has just had submitted to it as Item 1 of the proposed report on the American Telephone & Telegraph Company, prepared by one of the Commissioners, the adoption of a definitive position with respect to the oft criticized discrepancy between the telephone company's claims with respect to depreciation in operating expense and in valuation, and the excess of reserve balances over the existing depreciation in the property. The desperate financial situation of the railroads has afforded a striking demonstration of the error involved in determining net income and dividend payments year after year without providing reserves to record the losses in value which have undoubtedly occurred in the railroad properties, of the country as a result of deterioration, obsoleteness and the diversion of traffic to competing services. The gas and electric industries are faced with the immediate necessity of instituting depreciation accounting in lieu of retirement accounting, which they have generally practiced heretofore, and of providing reserves to represent existing depreciation in their properties—a step which may save them from the fate of the railroads, but which is bound to have a vital effect upon their balance sheets and income statements for years to come. The character and extent of such effect will depend upon determinations still to be made with respect to the definition of depreciation to be reflected in the accounts, and to whether the deficiency of existing reserves below required reserves is to be made up by transfer from existing surplus, or by charges to operating expense or to income over a period of years in the future.

It is hoped that the importance and urgency of these problems may provide some excuse for this discussion by an engineer, without training in the law, of some of the legal aspects of the problem of public utility depreciation.

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The history of the discussion of depreciation goes back nearly four hundred years. Professor A. C. Littleton calls attention in his book *Accounting Evolution to 1900* (American Institute Publishing Co., 1933) to a tract written in 1543 by Hugh Oldcastle, entitled *A Briefe Instruc-*

tion and Manner Hovv to Keepe Bookes of Accounts After the Order of Debitor and Creditor, revised and republished by John Mellis in 1588. From the latter edition he quotes a sample ledger entry on the credit side of the account "Implements of Householde" as follows:

Implements of householde here against is due to have xl.xs and is for so much as I doe finde at this day to be consumed and worn, which said xl.xs for the decay of said householde stuffe is borne to profit and losse in Debitor (15)	10	10	0

In the first three hundred and twenty-five years the discussion was largely confined to accounting treatises, although during the nineteenth century, frequent references to the subject commenced to appear in reports to railroad stockholders and other discussions of railway accounting in England and in the United States, with particular reference to its relation to the determination of net corporate income available for dividends.

The background of the present situation in American law, however, appears to be limited to the past sixty years, which, for the purpose of the present discussion, may conveniently be divided into three periods.

* * * * * * *

During the first of these periods, from 1876 to 1908, inclusive, the law in the United States would seem to have frowned upon any recognition of depreciation in advance of the replacement of property. Thus, in 1876, the Supreme Court of the United States said that "the public when referring to the profits of the business of a merchant, rarely, if ever, takes into account the depreciation of the buildings in which the business is carried on notwithstanding they may have been erected out of the capital invested." *Eyster v. Centennial Board of Finance, 94 U.S. 500 (1876).* And two years later, the Supreme Court supported the contention of the government that, in determining the net earnings of a railroad "Depreciation account, or expense not charged up **** the amount necessary to put the road in proper repair, but which was not actually expended for that purpose **** is not a proper charge. Only such expenditures as are actually made can with any propriety be claimed as a deduction from earnings." *U.S. v. Kansas Pacific Railway Co., 99 U.S. 455 (1878).*

In 1898, the word "depreciation" did not even appear in the decision of the Supreme Court in *Smyth v. Ames, 169 U.S. 466 (1898).* And in 1903, the Court found no constitutional objection to rates fixed to allow a return of six per cent on value, in addition to operating expenses and repairs, without "allowance for depreciation over and above the allow-

ance for repairs." *San Diego Land and Town Co. v. Jasper, 189 U.S. 439 (1903)*.

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In 1909, the law seems to this author to have been subjected to a major revision by the opinion of Mr. Justice Moody speaking for the United States Supreme Court in *City of Knoxville v. Knoxville Water Co., 212 U.S. 13 (1909)*. To quote a few extracts from this opinion: "Before coming to the question of profit at all, the company is entitled to earn a sufficient sum annually to provide not only for current repairs but for making good the depreciation and replacing the parts of the property when they come to the end of their lives. The company is not bound to see its property gradually waste, without making provision out of earnings for its replacement. ***

"It is not only the right of the company to make such a provision but it is its duty to its bond and stockholders and in the case of a public service corporation, at least, it is its plain duty to the public. *** If, however, a company fails to perform this plain duty and to exact sufficient returns to keep the investment unimpaired, whether this is the result of unwarranted dividends upon overissues of securities or of omission to exact proper prices for the output, the fault is its own."

* * * * * * *

The succeeding twenty-five years, from 1909 to 1934, constituting the second of our three historical periods, seems to this writer to have been characterized principally by a continuous controversy between the proponents of two equally erroneous interpretations of the Knoxville opinion. At the one extreme have been the supporters of straight line or sinking fund depreciation, contending that all losses due to depreciation occur continuously as a simple function of the passage of time, following either a straight line or a compound interest annuity curve. At the other extreme have been the supporters of the thesis that no loss due to depreciation occurs at all until property is retired from service, unless deferred maintenance exists, in which case the loss is limited to the cost to restore the property to good operating condition.

Between these extremes, a variety of intermediate positions have been urged, including the contention of the telephone industry that annual depreciation expense should be determined by the straight line method; but that, for valuation purposes, depreciation should be based upon the observed condition of the property, regardless of the growing discrep-

ancy between such observed depreciation and the reserves resulting from the continued application of straight line depreciation expense.

During this period, decisions of state public service commissions, state courts and the lower Federal Courts have appeared, from time to time, to support either one or the other of the extreme positions previously cited, as well as practically every intermediate position that has been urged. Some of the other highlights of the period have been the Report of the Special Committee on Valuation of Public Utilities of the American Society of Civil Engineers, *81 Trans. Am. Soc. C. E. 1311 (1917)*; the amendment of the Interstate Commerce Act authorizing the Interstate Commerce Commission to prescribe the classes of property upon which depreciation might be charged to operating expense, and to prescribe the percentages that might be charged on each such class, *Interstate Commerce Act, Sec. 20* (5), (*1920*); the adoption of retirement accounting designed only to equalize the effect of retirement losses recorded at the time of retirement in the Uniform Classification of Accounts for Gas and Electric Utilities, approved by the National Association of Railway and Utility Commissioners (1923); the hearings before the Interstate Commerce Commission under the amendment to the Interstate Commerce Act, the Commission's Report and Order, *118 I.C.C. 295* (*1926*) and the revised Report and Order of the Commission, *177 I.C.C. 351* (*1931*) favoring straight line depreciation; the periodic postponement of the effective date of the order of the Interstate Commerce Commission (continued down to the present year), *203 I.C.C. 81, 205 I.C.C. 33, 215 I.C.C. 629, etc.;* the report of the New York Legislative Commission on Revision of the Public Service Commissions Law, recommending that the Public Service Commission of New York be authorized to prescribe the method of determining and accounting for depreciation, *Report of Majority of Commission, Vol. 1, 36* (*1930*); the failure of the New York Legislature to approve this recommendation in sessions from 1930 to 1938 inclusive; the attempt of the New York Public Service Commission to prescribe straight line depreciation accounting in its new classifications of accounts, adopted in 1933, to be effective January 1, 1934; the disapproval of such attempted requirement by the Appellate Division of the Supreme Court of New York, *244 App. Div. 685* (*1935*), and by the Court of Appeals of New York, *271 N.Y. 103* (*1936*); and the publication of *A Review of Legal and Accounting Problems Relating to Depreciation,* by the Staff of the Public Service Commission of Wisconsin, favoring straight line depreciation (1933).

Throughout the resultant confusion and controversy the Supreme

Court of the United States seems to this writer to have developed the Knoxville decision gradually, logically and consistently. The opinion of Chief Justice Hughes, speaking for the majority of the court, in *Lindheimer v. Illinois Bell Telephone Co.*, *292 U.S. 151 (1934)*, is the culmination of a series of important decisions that afford the basis for what has since occurred, and what may be looked forward to in the immediate future.

The highlights of this development may be summarized as follows:

In ascertaining fair value from consideration of reproduction cost "the extent of existing depreciation should be shown and deducted." *Simpson v. Shepard, 230 U.S. 352, 456 (1913)*. Such depreciation is not that which has been taken care of by repairs and replacements, but is "the actual existing depreciation" *Simpson v. Shepard, 230 U.S. 352, 456 (1913)*. It is "the loss not restored by current maintenance which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence." *Lindheimer v. Illinois Bell Telephone Co., 292 U.S. 151, 167 (1934)*. In ascertaining such depreciation, examination by competent experts subsequent to the occurrence thereof, for the purpose of ascertaining existing facts, and estimates based on such examinations are preferable to averages and assumed probabilities. *Pacific Gas and Electric Co. v. San Francisco, 265 U.S. 403, 406 (1924)*. "The testimony of competent valuation engineers who examined the property and made estimates in respect of its condition is to be preferred to mere calculations based on averages and assumed probabilities." A deduction based on "straight line" calculation, without inspection of the property, cannot be approved. *McCardle v. Indianapolis Water Co., 272 U.S. 400, 416 (1926)*.

"Annual depreciation is the loss which takes place in a year." *Lindheimer v. Illinois Bell Telephone Co., 292 U.S. 151, 167 (1934)*. For valuation and rate purposes it must be based upon present value, and not upon original cost. *United Railways & Electric Co. of Baltimore v. West, 280 U.S. 234 (1930)*. For accounting purposes, however, it is the cost that must be recorded in the accounts, and which must eventually be charged off against operating revenues—whether by accumulation of a reserve through charges to operating expenses, by charge to profit and loss on retirement, or by a charge to current and future operating expense. *Kansas City Southern Railway Co. v. U.S., 231 U.S. 423, 448 (1913)*.

Such annual depreciation expense is a fact. *Georgia Railway & Power Co. v. Railroad Commission, 262 U.S. 625, 633 (1923)*; *Clark's Ferry Bridge Co. v. P.S.C., 291 U.S. 227 (1934)*. It need not be exactly the same

as the current charge for depreciation in the accounts. *Clark's Ferry Bridge Co. v. P.S.C., 291 U.S. 227 (1934)*. But if experience results in a balance in reserve unreasonably in excess of depreciation, the previous accounting charges will be considered excessive, and the excess reserve will be recognized, in fact, as "provision for capital additions, over and above the amount required for capital consumption." *Lindheimer v. Illinois Bell Telephone Co., 292 U.S. 174 (1934)*.

The property representing such excess reserves is nevertheless the property of the company, and consumers acquire no legal or equitable right therein. *Board of Commissioners v. New York Telephone Co., 271 U.S. 23, 31 (1926)*. Nor can such property or funds be used to justify rates otherwise confiscatory, or to reduce allowances for depreciation expense below actual current requirements. *Board of Commissioners v. New York Telephone Co., 271 U.S. 23, 31 (1926)*. The recognition of such ownership, however, does not make it necessary to permit excessive accumulation to continue after experience has shown them to be extravagant. *Smith v. Illinois Bell Telephone Co., 282 U.S. 133, 158 (1930); Clark's Ferry Bridge Co. v. P.S.C., 291 U.S. 227 (1934)*.

* * * * * * *

The third historical period, from 1934 up to and including the present, has been characterized by efforts of the Commissions and the State and Federal Courts to apply the principles summarized above as developed by the Supreme Court from the Knoxville case (1909) to the Lindheimer case (1934). A number of the Public Service Commissions have made efforts to harmonize determinations of annual and accrued depreciation in valuation and in accounting cases. *Louisiana P.S.C. v. So. Bell Telephone and Telegraph Co., 8 P.U.R. (N.S.) 1 (1935); Re Pacific Telephone and Telegraph Co. (Oregon) 8 P.U.R. (N.S.) 61 (1934); Re Northwestern Bell Telephone Co. (Nebraska) 5 P.U.R. (N.S.) 20 (1934); Re Northwestern Bell Telephone Co. (Nebraska) 11 P.U.R. (N.S.) 337 (1935); City of Memphis v. Southern Bell Telephone and Telegraph Co. (Tennessee) 6 P.U.R. (N.S.) 464 (1934); Re Southern Bell Telephone & Telegraph Co. (North Carolina) 7 P.U.R. (N.S.) 21 (1934); East Ohio Gas Co. v. City of Cleveland (Ohio) 4 P.U.R. (N.S.) 433 (1934)*; etc. And the lower Federal Courts and various State Courts have cited the Lindheimer case as authority for the two outstanding principles of the Supreme Court doctrine—that depreciation must be *actual;* and that in rate cases there must not be an unreasonable discrepancy between depreciation in valuation and in operating expense. *Chesa-*

peake & Potomac Telephone Co. v. West, 3 P.U.R. (N.S.) 241, 7 Fed. Supp. 214 (1934); Carey v. Corporation Commission, 5 P.U.R. (N.S.) 148, 168 Okla. 487, 33 P. (2d) 788 (1934); Ohio Bell Telephone Co. v. Public Utilities Comm., 15 P.U.R. (N.S.) 443, 131 Ohio St. 539, 3 N.E. (2d) 475 (1936); Cheltenham & Abingdon Sewerage Co. v. Public Service Comm., 15 P.U.R. (N.S.) 99, 122 Pa. Sup. Ct. 252, 186 Atl. 149 (1936); Southern Railway Co. v. Commissioner of Internal Revenue, 74 Fed. (2d) 890 (1935); etc.

The greatest impetus to the application of this doctrine, however, has resulted from the concerted effort of the regulatory commissions to harmonize accounting and valuation concepts for the purpose of making accounting records kept in the usual course of business more useful for regulatory purposes. The Federal Communications Commission (1935), the Federal Power Commission (1936), the New York Public Service Commission (1937) and numerous other State Public Service Commissions, led by the National Association of Railways and Utility Commissions (1936) have adopted new, uniform classifications of accounts requiring the reclassification of fixed capital accounts of utility companies on the basis of original cost to the person first devoting the property to public service; and depreciation accounting, in lieu of retirement accounting, on the basis of recording "as at the end of each month the estimated amount of depreciation accrued during that month."

The purpose of these new requirements was clearly stated by the New York Public Service Commission in its memorandum on Unification Plans, issued November 10, 1936, when it said:

> The time and expense required to investigate any company or system could be greatly reduced if continuing property records were kept up to date, if the books of account showed the original cost, *if the depreciation reserves represented even approximately the amount of depreciation actually existing in property* and if it were not necessary to estimate reproduction cost. (Italics supplied.)

These new developments have not completely ended the old controversy between the straight line and compound interest annuity enthusiasts on the one hand and the no-depreciation supporters of retirement accounting on the other. Their twenty-five year debate overlaps this new and forward looking era, largely in cases started a number of years ago. And within the year, the Appellate Division of the Supreme Court of New York has approved a rate order of the New York Public Service Commission based on straight line depreciation. *Long Island Lighting*

Company v. Maltbie, 18 P.U.R. (N.S.) 225, 292. This was in a case, however, where the company had presented estimates only of the cost to restore the property to efficient operating condition, and this writer does not disagree with the opinion of the court when it said:

> The petitioner contended that no depreciation should be deducted, simply the cost of restoring the depreciable property to a new condition. There was ample evidence before the Commission to sustain its method of determining depreciation.

It is not believed, however, that these vestigial remains of an ancient controversy between two erroneous theories will have an important influence on the working out of the doctrine of the Supreme Court as developed up through the Lindheimer case. The utility companies are alive to the necessity of developing a technique for the measurement of actual depreciation as it occurs, and of avoiding unreasonable disparities between actual depreciation and the reserves that result from experience with annual depreciation charges.

The solution of the problem is within the competence of their technical staffs, if they will now devote their energies and abilities to it, instead of continuing the futile attempt to support the thoroughly exploded contention that no loss occurs until property is retired, and that there is no depreciation in a well maintained property. And when a rational solution is presented to the courts, it is confidently predicted that it will receive judicial approval and that it will serve as the basis for the final rejection of the contention that all losses due to depreciation occur progressively with time in accordance with a straight line or a compound interest annuity formula. Let us briefly consider some of the general features of such a rational solution.

* * * * * * *

The recognition of depreciation as a fact embracing effects of deterioration, wear and tear, obsoleteness, inadequacy, change in demand, change in use and the requirement of public authority, seems to this writer to point the direction in which this rational solution, in accord with the doctrine of the Supreme Court, will be found. And if a further guide to its development is desired, nothing better can be recommended for study than the brilliant dissenting opinions of Justice Brandeis in *McCardle v. Indianapolis Water Co., 272 U.S. 400 (1926)*; *St. Louis and O'Fallon Ry. Co. v. U.S., 279 U.S. 461 (1929)*; and *United Railway and Electric Co. v. West, 280 U.S. 234 (1930)*. For while the force of these

opinions was lessened by the mistaken advocacy by Justice Brandeis of "prudent investment" as a basis both for valuation and depreciation, no one has raised more clearly than he the questions that need to be answered with respect to the nature of depreciation, its factual character, the distinction between the fact of depreciation and the depreciation charge in financial accounting, and the necessity of recognizing the effects of obsoleteness due to progress in the arts, changes in demand or in use due to the development of competitive services, to the movement of markets or to modification of operating programs, and of valid acts of public authority, as well as the effects of deterioration and wear and tear.

Indeed, Justice Brandeis' remarks, in his dissenting opinion in the West case, that none of the methods of depreciation accounting in use attempts "to determine the percentage of actual consumption of plant falling within a particular year, or within any period of years less than the service life"; and that no measure of the actual consumption of plant during a year has yet been invented, point directly to what is required to avoid unreasonable discrepancies between reserves and actual depreciation. They constitute a challenge to the engineering profession which has not been adequately met in the past.

For the engineer must abandon the assumption that depreciation is a simple function of cost, time and the interest rate; he must pay less attention to the shape of the curves and the forms of the equations for these functions, and more attention to the facts which they purport to explain.

In short, the engineer must discover measures of the actual consumption of plant, both at a particular time, and from year to year. He must work toward the development of methods of measuring all types of deterioration which will be as objective and as generally acceptable as the measurement of the consumption of fuel taken from a coal pile by its weight, using up of the material in a grindstone by its volume, or the wearing of the lead in a pencil by its length. Pending such complete development, he must establish all of the facts that can be established, so as to provide as sound a basis as possible for his engineering judgments expressed in percentages on a scale between 100 per cent representing condition new, and 0 per cent representing a condition requiring replacement.

The engineer must also develop better methods of measuring the effects of obsoleteness due to improvements in the art, by use of the analogy of decline in economic value equal to the difference between the totals of the cost of the equipment plus the sum of the present worths of the operating expenses (exclusive of depreciation or amortization) over a reason-

able period of time, for the existing equipment and for the most efficient improved equipment commercially available, respectively, where such improvement results in reduction of costs of production; and by the development of sounder bases of engineering judgment, expressed in percentages on a scale between 100 per cent and 0 per cent, where the improvement results in greater convenience, safety or reliability, or better quality of service.

The engineer must develop similar improved methods of measuring the effects of inadequacy, change in use and in demand, and the requirement of public authority. And finally he must develop methods of measuring, at least approximately, the amount of consumption of capital that occurs in a year, so that unreasonable discrepancies may be avoided between reserves and actual depreciation.

Such approximate measurement can be accomplished by recognizing that, if the actual depreciation at the beginning and at the end of a year could be determined, the actual depreciation loss during such year would be equal to the algebraic sum of the difference between such determined amounts of actual depreciation plus the net retirement loss during the year.

Of course, as a practical matter, the actual depreciation at the beginning and end of each year cannot be determined precisely for an extensive property, because of the cost in time and money that would be required. But when the actual depreciation at the beginning of a year has been arrived at, a sound engineering judgment as to the probable amount of actual depreciation at the end of the ensuing year can be arrived at by a competent engineer who has studied the property and who is familiar with the changes which it is undergoing during the year. In such determinations, the author has found, in his experience, that helpful guidance can be obtained from certain preliminary approximate computations.

Such preliminary approximate computations, it is suggested, can be helpfully based upon the assumption that deterioration will continue to progress for a single year directly in proportion to the passage of time, at a rate determined from the existing physical condition of the property, its age, and the effect of prior and prospective maintenance expenditures; and that all of the other causes of depreciation become effective and can be provided for as retirement losses in accordance with approved budgets for the ensuing year.

Such preliminary, approximate determinations, can be corrected on the basis of judgment as to the applicability of these assumptions to the particular property and the particular year, and allowance can be made

for known departures due to different incidence of deterioration, or to causes of obsoleteness, inadequacy, change in use or in demand, or requirement of public authority known to be effective on portions of the property during the year without justifying their retirement.

Such corrected determinations may also be reviewed and revised from time to time during the year, as experience permits the substitution of facts for predictions. And finally, if such a procedure is followed for a period of years, its correspondence with the facts can be checked, at intervals of five to ten years, by re-examination of the condition of the property, and by new engineering studies of the effects of obsoleteness, change in use or in demand, and requirement of public authority upon the property then remaining in use.

When such determinations of the facts with respect to depreciation have been made on the basis of sound engineering judgment, the depreciation expense charge in the accounts can then be fixed, as a matter of policy, either in strict accord with such determinations, or else so as to average the effect of changes in the rate of depreciation on the financial statements of a company over reasonable periods of years. In either case, such definition of policy will be based upon factual considerations, and not upon guesswork or financial manipulation; and the result will be the maintenance of that reasonable correspondence between reserves and actual depreciation which the Supreme Court has laid down as essential.

* * * * * * *

The experience of the author has convinced him that the procedure outlined is thoroughly practical and within the competence of trained engineering judgment. He believes that it affords a basis of co-operation between the engineering and the legal professions that may lead to a helpful solution of what has been one of the most vexing problems of public utility regulation.

In studies which he has made, the author has found that the result of the application of this concept of actual depreciation has been to produce amounts in depreciation reserves and amounts of annual depreciation expense intermediate between the extremes that result from the application of the erroneous straight line and retirement accounting theories. Perhaps this moderation of result is itself an indication of some inherent reasonableness in the method. In any case, further progress toward the truth will be facilitated by the carrying on of such co-operation between engineers and lawyers, as he has suggested, in that spirit of tolerance and fair play which Justice Holmes expressed when he said—speaking of the

interpretation of a franchise, but in simple words the spirit of which is equally applicable to the discussion of depreciation:

> An adjustment of this sort under a power to regulate rates has to steer between Scylla and Charybdis. On the one side, if the franchise is taken to mean that the most profitable return that could be got, free from competition, is protected by the 14th Amendment, then the power to regulate is null. On the other hand, if the power to regulate withdraws the protection of the Amendment altogether, then the property is naught. This is not a matter of economic theory, but of fair interpretation of a bargain. Neither extreme can have been meant. A midway between them must be hit. *Cedar Rapids Gas Light Co. v. Cedar Rapids, 223 U.S. 655, 669, 670.*

AN ASYMPTOTIC METHOD OF DETERMINING ANNUAL AND ACCRUED DEPRECIATION

BY JOSEPH JEMING

AN ASYMPTOTIC METHOD OF DETERMINING ANNUAL AND ACCRUED DEPRECIATION

By Joseph Jeming

Depreciation is a subject of major importance in the conduct of business. The close regulation to which public utilities are subject makes the proper treatment of depreciation of particular interest to them.

Accountants have devoted a great deal of time to the subject since depreciation is important in the preparation of financial statements. Engineers have made significant contributions to the problem, especially while engaged in the determination of "fair value" for public utility rate making. Economists have studied the problem by approaching it from the standpoint of economic value, while statisticians have attacked it mathematically.

The ample literature* on the subject makes it unnecessary to dwell at length upon the various methods of treating depreciation. In recent years, the straight-line† theory, which depends basically on estimates of probable service life, has been finding somewhat wider acceptance.‡

It is the purpose here to suggest a new way of estimating annual and accrued depreciation which will overcome some of the limitations of existing methods. The main object of this paper is the construction of a workable method of calculating depreciation so as to absorb the net retirement losses in terms of original investment over the useful life of the property which it represents. The basic considerations which govern the development of this proposed method are stated below.

1. The data required for proper application of the method must be easily available.

* National Association of Railroad and Utility Commissioners, *Report of Special Committee on Depreciation*, 1938; Marston and Agg, *Engineering Valuation*, McGraw Hill Book Company, 1936; James C. Bonbright, *Valuation of Property*, McGraw Hill Book Company, 1937; E. B. Kurtz, *The Science of Valuation and Depreciation*, Ronald Press, 1937; W. J. Graham, *Public Utility Valuation*, University of Chicago Press, 1934; "Statistical Analyses of Industrial Property Retirements," *Bulletin 125*, Iowa State College, 1935.

† *Financial Handbook*, Ronald Press, 1937, p. 382.

‡ Re Manchester Street R. Co., P.U.R. 1925D, 486; Michigan Bell Teleph. Co. v. Odell, P.U.R. 1931B, 192, 45 F (2d) 180; Re Citizens Teleph. Co., P.U.R. 1921E, 308; Federal Income Tax Practice—See *Treasury Bulletin "F"* (1931 ed.).

2. The volume of calculation should not be excessive nor too complex for practical application.

3. The assumptions implicit in the theory are not to be at variance with experience.

4. It must be possible to support the results empirically by comparison with the results obtained by other methods and in the light of experience.

The most important limitation upon any method of determining average service life is the lack of basic data. There are available, in almost all cases, figures representing the dollars of investment in classes of associated units of property which sometimes form fairly homogeneous groups. The figures reflect by years, over the history of the enterprise, the additions to investment, the retirements from the investment, and the investment balances at the beginning and end of each calendar year. The proposed method is based upon the utilization of these basic data for the development of the annual rates of depreciation.

The theory of this method is predicated upon the postulate that in the case of a homogeneous group of units of property, where the total number of units is kept constant, the ratio of retirements to the total in the group will approach some constant level.* Similarly, the ratio of additions or replacements to the total will approach the same limiting value. This is self-evident since in the case of a stable total, the replacements must be equal to the retirements. The limit of these annual ratios, which may be termed the retirement ratios and the addition ratios respectively, is equal to the reciprocal of average service life.† Thus, the limit is the annual depreciation rate required for depreciation accounting under the straight-line theory of depreciation.

In the case of a class of property represented by a fairly constant investment, the method of determining the average service life of the dollars of investment resolves itself into one of finding the limiting value of the retirement ratios. The average service life of the dollars is used because in accounting the dollars of investment are taken as the criterion rather than the units of property as such.

In order to find the limiting value of the retirement ratio, the observed ratios are fitted using the method suggested by the theory of Least Squares. The function selected must have the property of possessing a limiting value. The simplest of these functions is of the form:

* Re Community Nat. Gas Co. (1936) 15 P.U.R. (N.S.) 149.

† Gabriel A. D. Preinreich, "The Practice of Depreciation," *Econometrica*, July 1939, p. 259.

$$R = a + bx^{-1} + cx^{-2} \ldots$$

where R is the retirement ratio

and x is taken in years as units

The function has several favorable characteristics. It is analogous to an ordinary polynomial. The coefficients may be tested for the significance of their deviations from zero in order to determine objectively the number of terms to be used in fitting. The value of the first coefficient, a, is the desired value of the asymptote, that is the limiting value of the retirement ratios, as x approaches infinity. The estimate of variance of a may be found easily. It is comparatively simple to construct tables of x^{-n} and $S(x^{-n})$ for various ranges of x in years, for the purpose of setting up and solving the normal equations obtained by the Least Squares method.

Of course, it must be noted that the inferences as to the accuracy of the estimates of variance and therefore the reliability of the tests of significance depend upon the assumptions underlying the theory of Least Squares. Of these, an important assumption is that the data used must exhibit independence of their successive residuals from the fitted curve. In the case of public utility retirement ratios, as in the case of most economic data, it is usually difficult to find such independence of successive residuals. This means that in most cases the procedure used to find the number of terms in the fitted function yields distorted estimates of variance of the coefficients. However, this does not necessarily mean that the method is to be abandoned but that no undue reliance is to be placed on the results in the light of the theory of Least Squares.

Since all curve fitting is subject to errors of forecast, it is helpful to obtain the limiting value from another source which may serve as a check upon the results of the previous calculation. This may be accomplished by fitting the addition ratios by use of the function:

$$A = a' + b'x^{-1} + c'x^{-2} \ldots$$

where A is the addition ratio

In the case of a stable property, the limiting value of A, which in this case equals a', is equivalent to a previously determined and thus forms a check of the results. This check is particularly helpful in the practical application of the method where static properties are the exception rather than the rule, and the limiting values determined above are merely estimates of the true limits.

In actual practice, public utility property, when taken in terms of units

of dollars of investment, often exhibits growth. It is necessary, therefore, to take this factor into account. Growth of the investment or fixed capital balance may be due to either of two causes. The number of physical units of property may vary or the cost of the replacement units may change. Usually both factors play a part in causing growth. For accounting purposes, the dollars of original cost, that is, fixed capital, are to be recovered over the life of the property by charges against income. Under this consideration it is immaterial which factor causes growth of the fixed capital balance if the depreciation study be based on dollar data.

In the case of an increasing fixed capital balance, retirement ratios, for a property possessing a given average service life, are less in absolute value than for a static property. This follows because of the mortality dispersion over a period of years following the addition to fixed capital, which causes a lag in retirements. Similarly, the addition ratios are greater in absolute value. Thus, it follows that the limit of the observed retirement ratios is an underestimate of the true value, whereas, the limit of the addition ratios is an overestimate. Therefore, the true value of the reciprocal of the retirement and replacement ratio must lie between those two limits if it is assumed that conditions which prevailed in the past are representative in the long run.

Under these considerations, the position of the true value of the reciprocal of average service life, d, which must lie between the limit of R and the limit of A, depends on the function of the survivor curve of the property under consideration. This is illustrated in the following Table A for a few type curves assuming a uniform rate of growth of additions. The type curves used are selected from *Bulletin 125*, Iowa State College,* and adjusted one half year for simplicity of computation.

The use of any of the above mortality functions, in evaluating d, requires an implicit assumption which cannot be made on the basis of the available data. Furthermore, the complexity of most mortality functions makes a general solution for d in terms of a and a' impossible.

A practical solution of the problem may be developed by assuming the necessary conditions, such as a particular survivor function and a uniform rate of growth, which will make a rigid solution possible in some special cases; or by finding an empirical solution which will obviate the necessity of assuming ideal conditions. The main criticism of most other methods of determining average service life is the use of assumptions implicit in the theory which are at variance with the facts. It seems advis-

* *Op. cit.*

able, therefore, to choose the empirical solution which does not require the setting up of ideal conditions.

Averages of the two limiting values of A and R were tested empirically for reasonableness and compared with results obtained by the use of other methods. The geometric mean seemed to yield the best estimates. For example, the depreciation rates applicable to the dollars representing related groups of units of property of a large gas company, as developed by actuarial studies and engineering estimates, are compared with the geometric mean of a and a'.

Group	Depreciation Rate Other Methods	$\sqrt{a\,a'}$
Works and Station Structures	.025*	.020
Holders	.017*	.014
General Structures	.020*	.017
Boiler Plant Equipment	.050*	.042
Steam Engines	.050†	.053
Internal Combustion Engines	.038*	.033
Water Gas Sets	.045†	.050
Purification Apparatus	.025*	.048
Accessory Works Equipment	.029*	.027
Mains	.011†	.010
Services	.020†	.014
Meters	.027†	.027

* Based on Engineering Estimates.
† Based on Actuarial Study.

Note: For a detailed description of the property, see the Uniform System of Accounts, P.S.C., New York.

Table A on page 102 may be used to illustrate the reasonableness of the geometric mean of a and a' in hypothetical cases in which various type curves, various lives and various rates of growth are assumed.

The following Tables B, C, D and E are developed by use of Table A to show the estimates of d which result from the use of $\sqrt{a\,a'}$ and $\frac{a+a'}{2}$ compared with the true d on the basis of which Table A was constructed.

Of the two averages, that is, the geometric mean and the arithmetic mean of a and a', the $\sqrt{a\,a'}$ seems to yield the closer approximation to d.

TABLE A

True d	Uniform Rate of Growth	Limit of Retirement Ratios: Type S_1 Curve	Type R_4 Curve	Type L_3 Curve	Square Type Curve*
.0952	.02	.0883	.0871	.0875	.0865
.0952	.04	.0813	.0791	.0805	.0785
.0952	.07	.0726	.0687	.0710	.0676
.0952	.10	.0649	.0600	.0628	.0581
.0952	.15	.0545	.0479	.0513	.0449
.0952	.20	.0464	.0380	.0421	.0346
.0488	.02	.0410	.0402	.0410	.0399
.0488	.04	.0353	.0326	.0344	.0324
.0488	.07	.0280	.0247	.0264	.0233
.0488	.10	.0227	.0182	.0204	.0165
.0488	.15	.0165	.0111	.0134	.0091
.0488	.20	.0125	.0069	.0091	.0049
.0328	.02	.0256	.0244	.0251	.0241
.0328	.04	.0202	.0181	.0190	.0173
.0328	.07	.0149	.0114	.0129	.0102
.0328	.10	.0108	.0072	.0087	.0058
.0328	.15	.0071	.0034	.0047	.0021
.0328	.20	.0051	.0017	.0027	.0008
.0247	.02	.0177	.0168	.0172	.0162
.0247	.04	.0130	.0110	.0120	.0103
.0247	.07	.0085	.0058	.0070	.0048
.0247	.10	.0060	.0030	.0042	.0022
.0247	.15	.0037	.0012	.0018	.0005
.0247	.20	.0026	.0006	.0010	.0001
.0165	.02	.0102	.0091	.0096	.0086
.0165	.04	.0065	.0048	.0056	.0041
.0165	.07	.0037	.0018	.0024	.0012
.0165	.10	.0024	.0008	.0012	.0003
.0165	.15	.0014	.0003	.0004	.00003
.0165	.20	.0009	.000013	.0002	.000003

* Each unit retired exactly at average service life.

The range of rates of growth and true values of d assumed in these tables meets the characteristics of most public utility properties. The only exceptions are certain types of structures and conduit.

TABLE B

Type S_I Curve

Uniform Rate of Growth	Limit of *R*	True *d*	Geometric Mean	Arithmetic Mean
.02	.0883	.0952	.0977	.0983
.04	.0813	.0952	.0993	.1013
.07	.0726	.0952	.1017	.1076
.10	.0649	.0952	.1033	.1149
.15	.0545	.0952	.1056	.1295
.20	.0464	.0952	.1069	.1464
.02	.0410	.0488	.0499	.0510
.04	.0353	.0488	.0516	.0553
.07	.0280	.0488	.0524	.0630
.10	.0227	.0488	.0528	.0727
.15	.0165	.0488	.0524	.0915
.20	.0125	.0488	.0515	.1125
.02	.0256	.0328	.0342	.0356
.04	.0202	.0328	.0349	.0402
.07	.0149	.0328	.0355	.0499
.10	.0108	.0328	.0346	.0608
.15	.0071	.0328	.0334	.0821
.20	.0051	.0328	.0323	.1051
.02	.0177	.0247	.0258	.0277
.04	.0130	.0247	.0263	.0330
.07	.0085	.0247	.0258	.0435
.10	.0060	.0247	.0252	.0560
.15	.0037	.0247	.0238	.0787
.20	.0026	.0247	.0230	.1026
.02	.0102	.0165	.0176	.0202
.04	.0065	.0165	.0174	.0265
.07	.0037	.0165	.0165	.0387
.10	.0024	.0165	.0157	.0524
.15	.0014	.0165	.0145	.0764
.20	.0009	.0165	.0135	.1009

TABLE C

Type R_4 Curve

Uniform Rate of Growth	Limit of R	True d	Geometric Mean	Arithmetic Mean
.02	.0871	.0952	.0966	.0971
.04	.0791	.0952	.0971	.0991
.07	.0687	.0952	.0976	.1037
.10	.0600	.0952	.0980	.1100
.15	.0479	.0952	.0974	.1229
.20	.0380	.0952	.0951	.1380
.02	.0402	.0488	.0492	.0502
.04	.0326	.0488	.0487	.0526
.07	.0247	.0488	.0484	.0597
.10	.0182	.0488	.0464	.0682
.15	.0111	.0488	.0423	.0861
.20	.0069	.0488	.0378	.1069
.02	.0244	.0328	.0329	.0344
.04	.0181	.0328	.0324	.0381
.07	.0114	.0328	.0304	.0464
.10	.0072	.0328	.0277	.0572
.15	.0034	.0328	.0288	.0784
.20	.0017	.0328	.0185	.1017
.02	.0168	.0247	.0249	.0268
.04	.0110	.0247	.0236	.0310
.07	.0058	.0247	.0209	.0408
.10	.0030	.0247	.0176	.0530
.15	.0012	.0247	.0135	.0762
.20	.0006	.0247	.0110	.1006
.02	.0091	.0165	.0163	.0191
.04	.0048	.0165	.0147	.0248
.07	.0018	.0165	.0114	.0368
.10	.0008	.0165	.0090	.0508
.15	.0003	.0165	.0063	.0753
.20	.00001	.0165	.0014	.1000

TABLE D

Type L_3 Curve

Uniform Rate of Growth	Limit of R	True d	Geometric Mean	Arithmetic Mean
.02	.0875	.0952	.0970	.0975
.04	.0805	.0952	.0985	.1005
.07	.0710	.0952	.0997	.1060
.10	.0628	.0952	.1011	.1128
.15	.0513	.0952	.1016	.1263
.20	.0421	.0952	.1010	.1421
.02	.0410	.0488	.0500	.0510
.04	.0344	.0488	.0506	.0544
.07	.0264	.0488	.0504	.0614
.10	.0204	.0488	.0496	.0704
.15	.0134	.0488	.0468	.0884
.20	.0091	.0488	.0436	.1091
.02	.0251	.0328	.0336	.0351
.04	.0190	.0328	.0335	.0390
.07	.0129	.0328	.0327	.0479
.10	.0087	.0328	.0308	.0587
.15	.0047	.0328	.0270	.0797
.20	.0027	.0328	.0234	.1027
.02	.0172	.0247	.0253	.0272
.04	.0120	.0247	.0249	.0320
.07	.0070	.0247	.0232	.0420
.10	.0042	.0247	.0209	.0542
.15	.0018	.0247	.0165	.0768
.20	.0010	.0247	.0142	.1010
.02	.0096	.0165	.0169	.0196
.04	.0056	.0165	.0160	.0256
.07	.0024	.0165	.0132	.0374
.10	.0012	.0165	.0110	.0512
.15	.0004	.0165	.0078	.0754
.20	.0002	.0165	.0020	.1002

TABLE E

*Square Type Curve**

Uniform Rate of Growth	Limit of *R*	True *d*	Geometric Mean	Arithmetic Mean
.02	.0865	.0952	.0960	.0965
.04	.0785	.0952	.0964	.0985
.07	.0676	.0952	.0964	.1026
.10	.0581	.0952	.0958	.1081
.15	.0449	.0952	.0935	.1199
.20	.0346	.0952	.0901	.1346
.02	.0399	.0488	.0489	.0499
.04	.0324	.0488	.0484	.0524
.07	.0233	.0488	.0466	.0583
.10	.0165	.0488	.0438	.0665
.15	.0091	.0488	.0381	.0841
.20	.0049	.0488	.0317	.1049
.02	.0241	.0328	.0326	.0341
.04	.0173	.0328	.0315	.0373
.07	.0102	.0328	.0286	.0452
.10	.0058	.0328	.0248	.0558
.15	.0021	.0328	.0179	.0771
.20	.0008	.0328	.0127	.1008
.02	.0162	.0247	.0242	.0262
.04	.0103	.0247	.0228	.0303
.07	.0048	.0247	.0190	.0398
.10	.0022	.0247	.0150	.0522
.15	.0005	.0247	.0087	.0755
.20	.0001	.0247	.0045	.1001
.02	.0086	.0165	.0157	.0186
.04	.0041	.0165	.0134	.0241
.07	.0012	.0165	.0092	.0362
.10	.0003	.0165	.0055	.0503
.15	.00003	.0165	.0021	.0750
.20	.000003	.0165	.0008	.1000

* Each unit retired exactly at average service life.

In developing retirement and addition ratios from original data, a few other considerations are helpful. Usually original data are erratic in character and it has been found helpful to sacrifice some of the independence of the observed ratios by computing ratios of cumulated retirements to cumulated balances and cumulated additions to cumulated balances, over the observed range. The result is similar to the one obtained by a moving average in that the fluctuations in the raw data are minimized before fitting the asymptotic functions. This can be done only in those cases in which the entire history of additions and retirements is available. Should the historical data be available only for the more recent years, it is no longer possible to utilize this refinement of method and the simple retirement and addition ratios have to remain the basis.

It has been found that more reasonable results are obtained by fitting the retirement ratios over the range of observed values beginning with the first point at which the ratio is greater than zero. This conclusion has been reached empirically and is supported by the knowledge that the dearth of retirements in the first few years of the investment history is usually not representative of the mortality dispersion of physical property. The method is now summarized briefly before illustrating it by means of an example taken from its extensive application to public utility properties.

The function $R = a + bx^{-1} + cx^{-2}$ . . . is fitted to the retirement ratios by Least Squares.

The function $A = a' + b'x^{-1} + c'x^{-2}$ . . . is fitted to the addition ratios.

The estimate of the true value of the reciprocal of average service life, d, is found by the equation:

$$d = \sqrt{a\,a'}$$

$$\text{Average Service Life} = \frac{1}{d}$$

The choice of the number of constants in the asymptotic functions depends on several considerations.

Those functions which result in negative values of a or a' must be eliminated. Those functions containing coefficients which were not significantly different from zero, under the criterion of $t = 2$, were not used. The symbol t is the ratio of the coefficient to its standard error.

Furthermore, it has not been found practical to use functions containing more than four constants in any event. Usually the above criteria have

succeeded in limiting the solution to one value of a and a'. In those cases, however, where more than one value resulted, the values of a and a' nearest each other were chosen since the theory is based on the premise that the two values are equal in the case of a static property.

The rate d is used for the purpose of computing annual depreciation to be credited to the reserve account over the history of the fixed capital account. Since the dollars of retirements are available in the original data, it is a simple matter to build up the reserve for depreciation to the required point in time.

Salvage may be taken care of in the usual manner by adjusting the annual depreciation rate accordingly and crediting the reserve for salvage realized.

In the following illustration of the method, the so-called Outside Plant accounts of a large urban electric company are analyzed. The greater portion of the investment in these associated accounts represents underground and overhead conductors, transformers, poles and fixtures.

In building up the reserve for depreciation from the inception of the property, an estimated depreciation rate must be used because no experience is available at the beginning. Therefore, an average life of 20 years and a realizable net salvage of 20 per cent is assumed. This results in the application of a net depreciation rate of 5 per cent $\times$ .80 or 4 per cent.

At the end of the 14th year, enough experience has accrued so that the asymptotic method may be used to check the original life estimate. The depreciation rate of 5.65 per cent is developed in Tables V-A and VI-A, and illustrated in Chart A. This rate, when combined with a salvage factor of 9 per cent based on experience, results in a net depreciation rate of 5.14 per cent, which is used thereafter until the 28th year. At this time a new estimate is developed by the use of a ten-year band from the 19th to the 28th year inclusive. This is illustrated in Chart B and developed in Tables V-B and VI-B.

Similarly, a new estimate is made in the 36th year using the ten-year experience from the 26th to the 35th year, inclusive. This is shown in Chart C and Tables V-C and VI-C.

The use of bands makes it possible to build up reserves on the basis of estimates which actually could have been made at the points indicated. The use of cumulative ratios gives some weight to all prior experience but the use of bands gives greater weight to the more recent experience.

CHART A

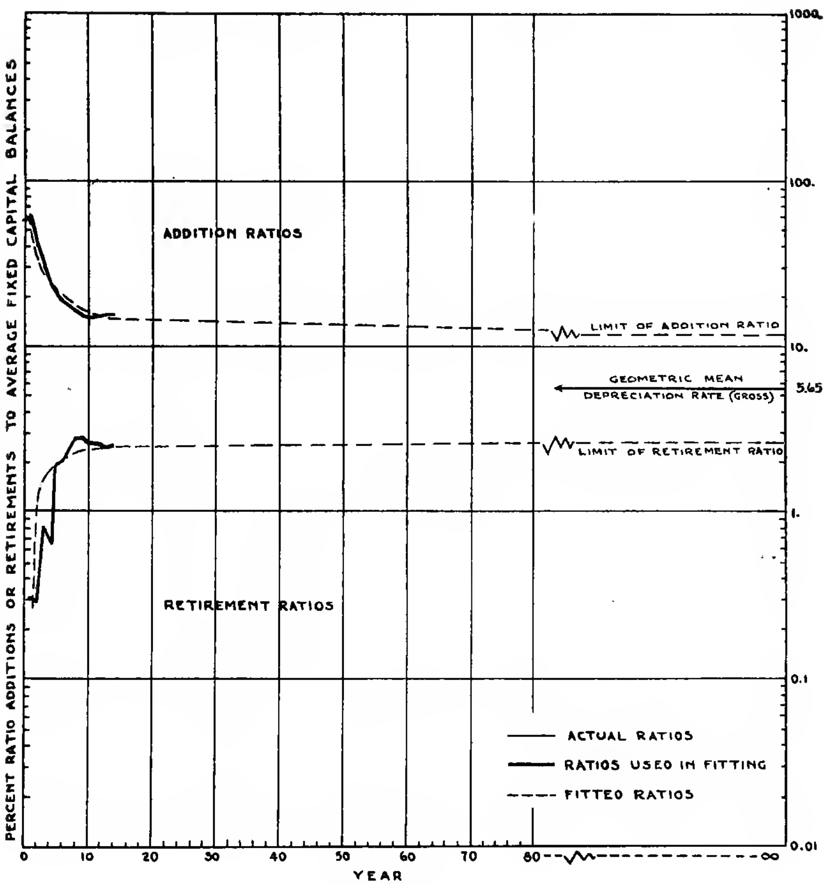
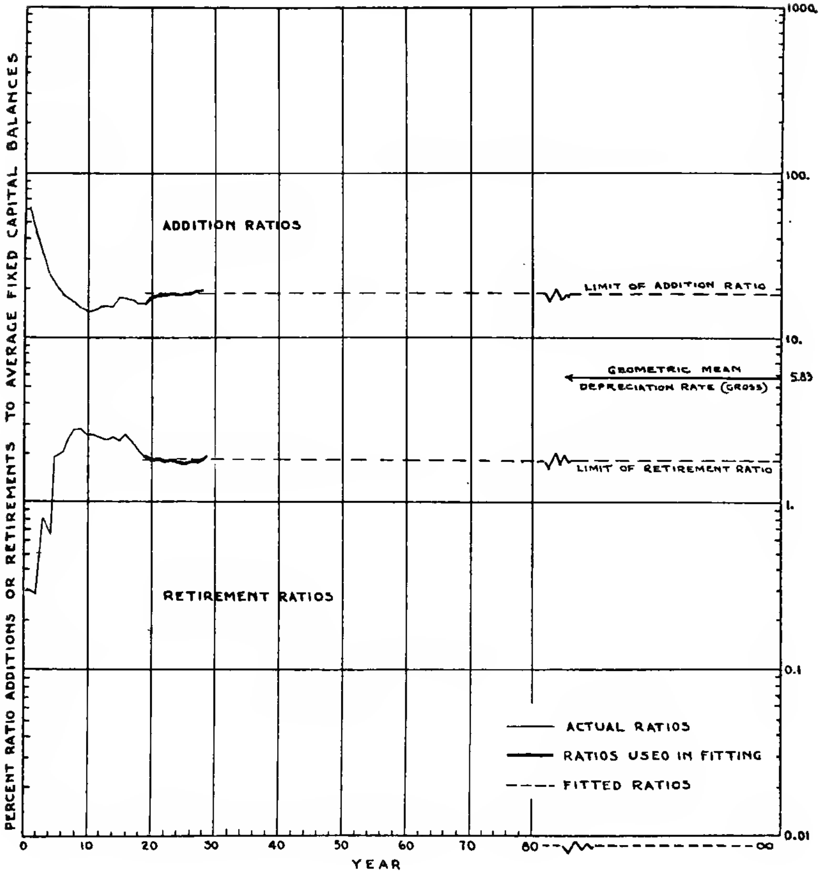

PERCENT RATIO ADDITIONS OR RETIREMENTS TO AVERAGE FIXED CAPITAL BALANCES
ADDITION RATIOS
RETIREMENT RATIOS
LIMIT OF ADDITION RATIO
GEOMETRIC MEAN
DEPRECIATION RATE (GROSS)
5.65
LIMIT OF RETIREMENT RATIO
1000.
100.
10.
1.
0.1
0.01
ACTUAL RATIOS
RATIOS USED IN FITTING
FITTED RATIOS
0
10
20
30
40
50
60
70
80
∞
YEAR

CHART B



PERCENT RATIO ADDITIONS OR RETIREMENTS TO AVERAGE FIXED CAPITAL BALANCES
ADDITION RATIOS
LIMIT OF ADDITION RATIO
GEOMETRIC MEAN DEPRECIATION RATE (GROSS)
5.83
LIMIT OF RETIREMENT RATIO
RETIREMENT RATIOS
ACTUAL RATIOS
RATIOS USED IN FITTING
FITTED RATIOS
1000.
100.
10.
1.
0.1
0.01
0
10
20
30
40
50
60
70
80
∞
YEAR

CHART C

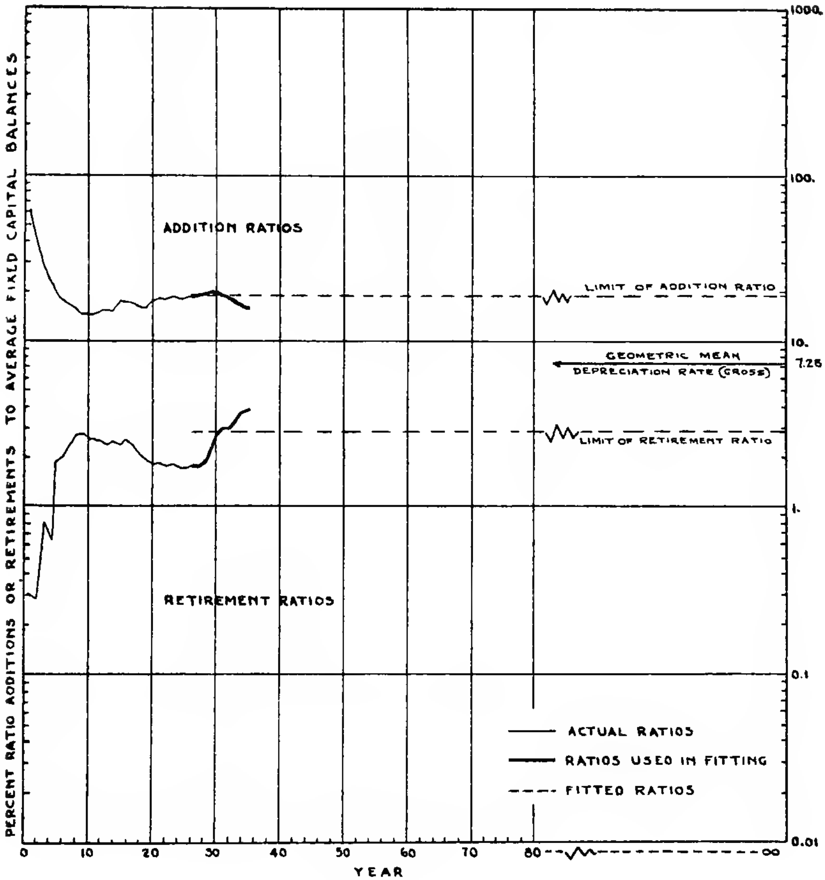

PERCENT RATIO ADDITIONS OR RETIREMENTS TO AVERAGE FIXED CAPITAL BALANCES
ADDITION RATIOS
LIMIT OF ADDITION RATIO
GEOMETRIC MEAN DEPRECIATION RATE (GROSS)
7.26
LIMIT OF RETIREMENT RATIO
RETIREMENT RATIOS
ACTUAL RATIOS
RATIOS USED IN FITTING
FITTED RATIOS
1000.
100.
10.
1.
0.1
0.01
0
10
20
30
40
50
60
70
80
∞
YEAR

TABLE I

Basic Data Available from Fixed Capital Record

ACCOUNTS NOS. 331–342—OUTSIDE PLANT (EXCLUSIVE OF METERS, METER INSTALLATION AND CONDUIT)

Year (1)	Balance at Beginning of the Year (2)	Additions During the Year (3)	Retirements During the Year (4)	Balance at the End of the Year (5)	Average Balance (6)	Net Retirement Loss During the Year (7)
1900, Aug. 1	$ 352,120.80	$ 23,703.69	$	$ 375,824.49	$ 363,972.64	$ ....
1901, Jan. 1	375,824.49	89,287.93	2,341.11	462,771.31	419,297.90	2,341.11
1902	462,771.31	54,588.02	1,405.72	515,953.61	489,362.46	1,405.72
1903	515,953.61	23,229.11	11,046.81	528,135.91	522,044.76	11,046.81
1904	528,135.91	22,627.92	421.29	550,342.54	539,239.22	421.29
1905	550,342.54	37,641.58	41,566.34	546,417.78	548,380.16	41,566.34
1906	546,417.78	58,479.57	19,057.78	585,839.57	566,128.68	19,057.78
1907	585,839.57	56,621.39	24,129.97	618,330.99	602,085.28	24,129.97
1908	618,330.99	64,765.75	36,492.17	646,604.57	632,467.78	36,492.17
1909	646,604.57	71,117.45	12,314.13	705,407.89	676,006.23	10,572.89
1910	705,407.89	114,893.43	9,065.25	811,236.07	758,321.98	6,239.57
1911	811,236.07	168,059.91	22,384.13	956,911.85	884,073.96	20,459.53
1912	956,911.85	174,857.45	18,795.51	1,112,973.79	1,034,942.82	16,667.60
1913	1,112,973.79	255,733.35	23,507.13	1,345,200.01	1,229,086.90	19,396.19
1914	1,345,200.01	237,155.74	53,399.99	1,528,955.76	1,437,077.88	46,224.64
1915	1,528,955.76	493,037.87	17,873.21	2,004,120.42	1,766,538.09	15,379.74
1916	2,004,120.42	324,487.31	95,389.38	2,233,218.35	2,118,669.39	66,766.12
1917	2,233,218.35	312,154.54	21,119.34	2,524,253.55	2,378,735.95	17,419.85
1918	2,524,253.55	205,315.91	13,498.02	2,716,071.44	2,620,162.50	5,231.14
1919	2,716,071.44	517,549.79	24,535.91	3,209,085.32	2,962,578.38	14,903.78
1920	3,209,085.32	1,041,593.44	27,323.30	4,223,355.46	3,716,220.39	17,424.14
1921	4,223,355.46	976,044.82	96,406.65	5,102,993.63	4,663,174.54	69,285.31
1922	5,102,993.63	1,055,451.05	82,264.98	6,076,179.70	5,589,586.67	57,746.23
1923	6,076,179.70	1,320,671.14	114,682.21	7,282,168.63	6,679,174.16	97,692.01
1924	7,282,168.63	1,471,242.13	143,723.24	8,609,687.52	7,945,928.08	113,362.83
1925	8,609,687.52	2,001,328.92	148,533.88	10,462,482.56	9,536,085.04	116,142.42
1926	10,462,482.56	2,485,806.46	232,699.68	12,715,589.34	11,589,035.95	187,734.52
1927	12,715,589.34	3,171,087.52	283,645.06	15,603,031.80	14,159,310.57	239,839.92
1928	15,603,031.80	3,507,331.19	561,826.42	18,548,536.57	17,075,784.18	461,250.72
1929	18,548,536.57	4,604,576.48	913,059.94	22,240,053.11	20,394,294.84	766,794.98
1930	22,240,053.11	3,600,017.50	1,173,077.15	24,666,993.46	23,453,523.28	1,078,934.79
1931	24,666,993.46	2,840,478.29	1,145,684.92	26,361,786.83	25,514,390.14	1,056,918.84
1932	26,361,786.83	3,895,506.82	963,281.04	29,294,012.61	27,827,899.72	848,656.42
1933	29,294,012.61	2,762,806.19	1,871,270.57	30,185,548.23	29,739,780.42	1,187,813.37
1934	30,185,548.23	2,493,077.58	1,891,154.38	30,787,471.43	30,486,509.83	1,163,028.26
1935	30,787,471.43	3,836,069.57	1,512,044.43	33,111,496.57	31,949,484.00	1,278,795.13
1936	33,111,496.57	3,494,825.65	1,412,293.06	35,194,029.16	34,152,762.86	1,115,517.47
1937	35,194,029.16	2,854,383.64	960,712.49	37,087,700.31	36,140,864.74	692,838.37
1938	37,087,700.31					

The first five columns of Table I show the basic data upon which the depreciation study is predicated. Column (6) is easily computed by averaging Columns (2) and (5).

Table II illustrates the development of cumulative ratios. These are used because the entire history is available in this case and retirements are erratic as indicated in Column (4) of Table I.

TABLE II

Computation of Cumulative Ratios

Cumulative Additions to Cumulated Average Balances
Cumulative Retirements to Cumulative Average Balances

Year	X (1)	Cumulative Average Balance (2)	Cumulative Additions (3)	Cumulative Retirements (4)	Cumulative Addition Ratio (5)	Cumulative Retirement Ratio (6)
1900	1	$ 363,972.64	$ 375,824.49	$		
1901	2	783,270.54	465,112.42	2,341.11	.5938	.0030
1902	3	1,272,633.00	519,700.44	3,746.83	.4084	.0029
1903	4	1,794,677.76	542,929.55	14,793.64	.3025	.0082
1904	5	2,333,916.98	565,557.47	15,214.93	.2423	.0065
1905	6	2,882,297.14	603,199.05	56,781.27	.2093	.0197
1906	7	3,448,425.82	661,678.62	75,839.05	.1919	.0220
1907	8	4,050,511.10	718,300.01	99,969.02	.1773	.0247
1908	9	4,682,978.88	783,065.76	136,461.19	.1672	.0291
1909	10	5,358,985.11	854,183.21	148,775.32	.1594	.0278
1910	11	6,117,307.09	969,076.64	157,840.57	.1584	.0258
1911	12	7,001,381.05	1,137,136.55	180,224.70	.1624	.0257
1912	13	8,036,323.87	1,311,994.00	199,020.21	.1633	.0248
1913	14	9,265,410.77	1,567,727.35	222,527.34	.1692	.0240
1914	15	10,702,488.65	1,804,883.09	275,927.33	.1686	.0258
1915	16	12,469,026.74	2,297,920.96	293,800.54	.1843	.0236
1916	17	14,587,696.13	2,622,408.27	389,189.92	.1798	.0267
1917	18	16,966,432.08	2,934,562.81	410,309.26	.1730	.0242
1918	19	19,586,594.58	3,139,878.72	423,807.28	.1603	.0216
1919	20	22,549,172.96	3,657,428.51	448,343.19	.1622	.0199
1920	21	26,265,393.35	4,699,021.95	475,666.49	.1789	.0181
1921	22	30,928,567.89	5,675,066.77	572,073.14	.1835	.0185
1922	23	36,518,154.56	6,730,517.82	654,338.12	.1843	.0179
1923	24	43,197,328.72	8,051,188.96	769,020.33	.1864	.0178
1924	25	51,143,256.80	9,522,431.09	912,743.57	.1862	.0178
1925	26	60,679,341.84	11,523,760.01	1,061,277.45	.1899	.0175
1926	27	72,268,377.79	14,009,566.47	1,293,977.13	.1939	.0179
1927	28	86,427,688.36	17,180,653.99	1,577,622.19	.1988	.0183
1928	29	103,503,472.54	20,687,985.18	2,139,448.61	.1999	.0207
1929	30	123,897,767.38	25,292,561.66	3,052,508.55	.2041	.0246
1930	31	147,351,290.66	28,892,579.16	4,225,585.70	.1961	.0287
1931	32	172,865,680.80	31,733,057.45	5,371,270.62	.1836	.0311
1932	33	200,693,580.52	35,628,564.27	6,334,551.66	.1775	.0316
1933	34	230,433,360.94	38,391,370.46	8,205,822.23	.1666	.0356
1934	35	260,919,870.77	40,884,448.04	10,096,976.61	.1567	.0387
1935	36	292,869,354.77	44,720,517.61	11,609,021.04	.1527	.0396
1936	37	327,022,117.63	48,215,343.26	13,021,314.10	.1474	.0398
1937	38	363,162,982.37	51,069,726.90	13,982,026.59	.1406	.0385

The normal equations for the function $A = a' + b'x^{-1} + c'x^{-2}$ are developed for the method of Least Squares. These are:

$$
\begin{aligned}
n'a' + b'S(x^{-1}) + c'S(x^{-2}) \quad &= S(A) \\
a'S(x^{-1}) + b'S(x^{-2}) + c'S(x^{-3}) &= S(Ax^{-1}) \\
a'S(x^{-2}) + b'S(x^{-3}) + c'S(x^{-4}) &= S(Ax^{-2})
\end{aligned}
$$

where n' is number of observed ratios and
S indicates the sum of successive values indicated.

The coefficients of the constants a', b' and c' in the left-hand members of the above equations are found directly from tables of $S(x^{-n})$ compiled for this purpose.

The right-hand members of the equations are computed in Columns (2), (3) and (4) of Table III.

Similarly, the values for the solution of the normal equations of the function $R = a + bx^{-1}$ are developed in Table IV.

TABLE III

Analysis of Additions

COMPUTATION OF VALUES FOR NORMAL EQUATIONS

X	Cumulative Addition Ratio A	Ax^{-1}	Ax^{-2}	Ax^{-3}	A^2	Sum	Check
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	.5938	.5938000	.5938000	.5938000	.3525984	4.5938000	2.7277984
2	.4084	.2042000	.1021000	.0510500	.1667906	2.2834000	.9325406
3	.3025	.1008333	.0336111	.0112037	.0915063	1.7839814	.5396544
4	.2423	.0605750	.0151438	.0037859	.0587093	1.5704250	.3805140
5	.2093	.0418600	.0083720	.0016744	.0438065	1.4573000	.3050129
6	.1919	.0319833	.0053306	.0008884	.0368256	1.3909741	.2669279
7	.1773	.0253286	.0036184	.0005169	.0314353	1.3434808	.2381991
8	.1672	.0209000	.0026125	.0003266	.0279558	1.3097781	.2189949
9	.1594	.0177111	.0019679	.0002186	.0254084	1.2842285	.2047060
10	.1584	.0158400	.0015840	.0001584	.0250905	1.2694000	.2010730
11	.1624	.0147636	.0013422	.0001220	.0263738	1.2623249	.2050016
12	.1633	.0136083	.0011340	.0000945	.0266669	1.2541564	.2048037
13	.1692	.0130154	.0010012	.0000770	.0286286	1.2524955	.2119222
14	.1686	.0120429	.0008602	.0000614	.0284260	1.2454950	.2099905
(Sub-Total 1901–1914)	(3.2740)	(1.1664615)	(.7724779)	(.6639778)	(.9702220)		(6.8471392)
15	.1843	.0122867	.0008191	.0000546	.0339665	1.2557074	.2314269
16	.1798	.0112375	.0007024	.0000439	.0323280	1.2464504	.2241118
17	.1730	.0101765	.0005986	.0000352	.0299290	1.2354872	.2137393
18	.1603	.0089056	.0004947	.0000275	.0256961	1.2191135	.1954239
19	.1622	.0085368	.0004493	.0000236	.0263088	1.2177475	.1975186
20	.1789	.0089450	.0004473	.0000224	.0320052	1.2315250	.2203198
21	.1835	.0087381	.0004161	.0000198	.0336723	1.2334947	.2263463
22	.1843	.0083773	.0003808	.0000173	.0339665	1.2319145	.2270418
23	.1864	.0081044	.0003524	.0000153	.0347450	1.2318509	.2296170
24	.1862	.0077583	.0003233	.0000135	.0346704	1.2296751	.2289655
25	.1899	.0075960	.0003038	.0000122	.0360620	1.2315640	.2338740
26	.1939	.0074577	.0002868	.0000110	.0375972	1.2338977	.2392528
27	.1988	.0073630	.0002727	.0000101	.0395214	1.2372595	.2459672
28	.1999	.0071393	.0002550	.0000091	.0399600	1.2369354	.2472634
(Sub-Total 1919–1928)	(1.8640)	(.0800159)	(.0034875)	(.0001543)	(.3485088)		(2.2961664)
29	.2041	.0070379	.0002427	.0000084	.0416568	1.2398129	.2530458
30	.1961	.0065367	.0002179	.0000073	.0384552	1.2305814	.2413170
31	.1836	.0059226	.0001911	.0000062	.0337090	1.2169323	.2234288
32	.1775	.0055469	.0001733	.0000054	.0315063	1.2097571	.2147319
33	.1666	.0050485	.0001530	.0000046	.0277556	1.1978491	.1995617
34	.1567	.0046088	.0001356	.0000040	.0245549	1.1870023	.1860033
35	.1527	.0043629	.0001246	.0000036	.0233173	1.1821110	.1805083
(Sub-Total 1926–1935)	(1.8299)	(.0610243)	(.0020527)	(.0000697)	(.3380337)		(2.2310802)
36	.1474	.0040944	.0001137	.0000032	.0217268	1.1759708	.1733381
37	.1406	.0038000	.0001027	.0000028	.0197684	1.1683772	.1642738

COMPUTATION OF VALUES FOR NORMAL EQUATIONS

X	Cumulative Retirement Ratio R	Rx^{-1}	Rx^{-2}	Rx^{-3}	R^2	Sum	Check
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	.0030	.0030000	.0030000	.0030000	.0000090	4.0030000	.0120090
2	.0029	.0014500	.0007250	.0003625	.0000084	1.8779000	.0054459
3	.0082	.0027333	.0009111	.0003037	.0000672	1.4896814	.0122154
4	.0065	.0016250	.0004063	.0001016	.0000423	1.3346250	.0086751
5	.0197	.0039400	.0007880	.0001576	.0003881	1.2677000	.0249737
6	.0220	.0036667	.0006111	.0001019	.0004840	1.2210741	.0268636
7	.0247	.0035286	.0005041	.0000720	.0006101	1.1908808	.0294148
8	.0291	.0036375	.0004547	.0000568	.0008468	1.1716781	.0340958
9	.0278	.0030889	.0003432	.0000381	.0007728	1.1526285	.0320431
10	.0258	.0025800	.0002580	.0000258	.0006656	1.1368000	.0293294
11	.0257	.0023364	.0002124	.0000193	.0006605	1.1256249	.0289286
12	.0248	.0020667	.0001722	.0000144	.0006150	1.1156564	.0276683
13	.0240	.0018462	.0001420	.0000109	.0005760	1.1072955	.0265751
14	.0258	.0018429	.0001316	.0000094	.0006656	1.1026950	.0284495
(Sub-Total 1901–1914)	(.2700)	(.0373422)	(.0086597)	(.0042740)	(.0064114)		(.3266873)
15	.0236	.0015733	.0001049	.0000070	.0005570	1.0950074	.0258422
16	.0267	.0016688	.0001043	.0000065	.0007129	1.0933504	.0291925
17	.0242	.0014235	.0000837	.0000049	.0005856	1.0866872	.0262978
18	.0216	.0012000	.0000667	.0000037	.0004666	1.0804135	.0233369
19	.0199	.0010474	.0000551	.0000029	.0003960	1.0754475	.0214014
20	.0181	.0009050	.0000453	.0000023	.0003276	1.0707250	.0193801
21	.0185	.0008810	.0000420	.0000020	.0003423	1.0684947	.0197672
22	.0179	.0008136	.0000370	.0000017	.0003204	1.0655145	.0190727
23	.0178	.0007739	.0000336	.0000015	.0003168	1.0632509	.0189259
24	.0178	.0007417	.0000309	.0000013	.0003168	1.0612751	.0188907
25	.0175	.0007000	.0000280	.0000011	.0003063	1.0591640	.0185354
26	.0179	.0006885	.0000265	.0000010	.0003204	1.0578977	.0189364
27	.0183	.0006778	.0000251	.0000009	.0003349	1.0567549	.0193386
28	.0207	.0007393	.0000264	.0000009	.0004285	1.0577354	.0218951
(Sub-Total 1919–1928)	(.1844)	(.0079682)	(.0003499)	(.0000156)	(.0034100)		(.1961435)
29	.0246	.0008483	.0000293	.0000010	.0006052	1.0603129	.0260837
30	.0287	.0009567	.0000319	.0000011	.0008237	1.0631814	.0305133
31	.0311	.0010032	.0000324	.0000010	.0009672	1.0644323	.0331038
32	.0316	.0009875	.0000309	0000010	.0009986	1.0638571	.0336179
33	.0356	.0010788	.0000327	.0000010	.0012674	1.0668491	.0379798
34	.0387	.0011382	.0000335	.0000010	.0014977	1.0690023	.0413704
35	.0396	.0011314	.0000323	.0000009	.0015682	1.0690110	.0423328
(Sub-Total 1926–1935)	(.2868)	(.0092497)	(.0003010)	(.0000098)	(.0088118)		(.3051718)
36	.0398	.0011056	.0000307	.0000009	.0015840	1.0683708	.0425212
37	.0385	.0010405	.0000281	.0000008	.0014823	1.0662772	.0410517

TABLE V-A

Analysis of Additions—Solution of Normal Equations

Function: $A = a' - b'x^{-1}$

Range: 1–14 $\qquad n' = 14$

a'	b'	c'				l
			1.0000000			
				1.0000000		
					1.0000000	
1.0000000						
1.0000000						
1.0000000						
	1.0000000					
	1.0000000					
		c'				
		1.0000000				
		1.0000000				
		1.0000000				
14.0000000	3.2515623		1.0000000			3.2740000
3.2515623	1.5759959			1.0000000		1.1664615
1.0000000	.2322545		.0714286			.2338571
1.0000000	.4846088			.3075445		.3597388
	−.2524343		.0714286	−.3075445		−.1248817
	b'		−.2829592	1.2183150		.4947097
4.3056226	1.0000000		.3075445			1.0069006
2.0631794	1.0000000			.6345194		.7401425
2.2424432			.3075445	−.6345194		.2667581
a'			.1371471	−.2829590		.1189587

Use $a' = .1190$

$$s^2 = \frac{1}{n' - r}\left[S(A^2) - a'l_1 - b'l_2 - c'l_3\right] = .0003076$$

$$s^2 = \frac{1}{12}\left[.9702220 - .3894708 - .5770598\right] = \frac{1}{12}\left[.0036914\right]$$

$$s_{a'}^2 = s^2c_{11} = (.0003076)(.1371471) = .0000422$$

$$s_{b'}^2 = s^2c_{22} = (.0003076)(1.2183150) = .0003748$$

$$s_{a'} = .0065 \qquad t_{a'} = a' \div s_{a'} = 18.30$$

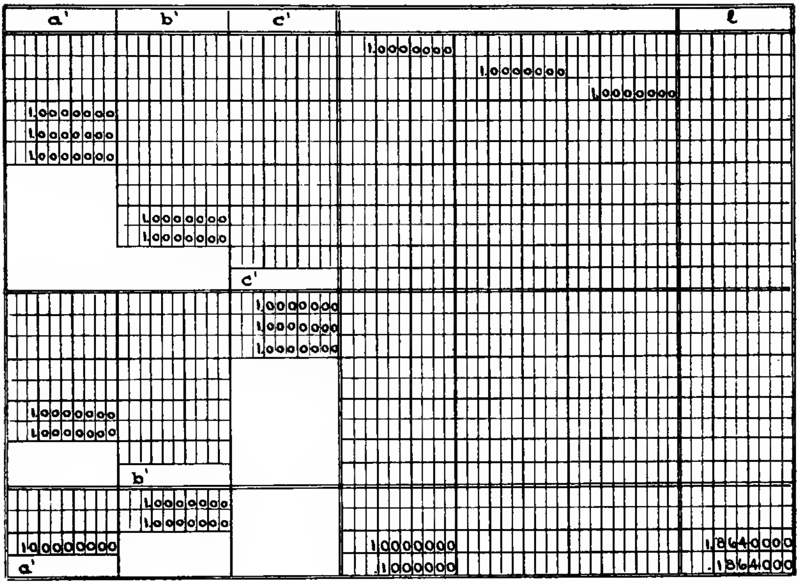
$$s_{b'} = .0194 \qquad t_{b'} = b' \div s_{b'} = 25.50$$

TABLE V-B

Analysis of Additions—Solution of Normal Equations

Function: $A = a'$

Range: 19–28 *$n' = 10$*



Use $a' = .1864$

$$s^2 = \frac{1}{n' - r}\left[S(A^2) - a'l_1 - b'l_2 - c'l_3\right] = .0001177$$

$$s^2 = \frac{1}{9}\left[.3485088 - .3474496\right] = \frac{1}{9}\left[.0010592\right]$$

$$s_{a'}^2 = s^2 c_{11} = (.0001177)(.1000000) = .0000118$$

$$s_{a'} = .0034 \qquad t_{a'} = a' \div s_{a'} = 54.82$$

TABLE V-C

Analysis of Additions—Solution of Normal Equations

Function: $A = a'$

Range: 26–35 $n' = 10$

a	b	c				l
			1.0000000			
				1.0000000		
					1.0000000	
1.0000000						
1.0000000						
1.0000000						
	1.0000000					
	1.0000000					
		c				
		1.0000000				
		1.0000000				
		1.0000000				
1.0000000						
1.0000000						
	b					
	1.0000000					
	1.0000000					
10.0000000			1.0000000			1.8299000
a			.1000000			.1829900

Use $a' = .1830$

$$s^2 = \frac{1}{n' - r}\left[S(A^2) - a'l_1 - b'l_2 - c'l_3\right] = .0003534$$

$$s^2 = \frac{1}{9}\left[.3380337 - .3348534\right] = \frac{1}{9}\left[.0031803\right]$$

$$s_{a'}^2 = s^2 c_{11} = (.0003534)(.1000000) = .0000353$$

$$s_{a'} = .0059 \qquad t_{a'} = a' \div s_{a'} = 31.02$$

TABLE VI-A

Analysis of Retirements—Solution of Normal Equations

Function: $R = a - bx^{-1}$

Range: 1–14 $n' = 14$

a	b	c				l
			1.0000000			
				1.0000000		
					1.0000000	
1.0000000						
1.0000000						
1.0000000						
	1.0000000					
	1.0000000					
		c				
		1.0000000				
		1.0000000				
		1.0000000				
14.0000000	3.2515623		1.0000000			.2700000
3.2515623	1.5759959			1.0000000		.0373422
1.0000000	.2322545		.0714286			.0192857
1.0000000	.4846888			.3075445		.0114844
	−.2524343		.0714286	−.3075445		.0078013
	b		−.2829592	1.2183150		−.0309043
4.3056226	1.0000000		.3075445			.0830370
2.0631794	1.0000000			.6345194		.0236944
2.2424432			.3075445	−.6345194		.0593426
a			.1371471	−.2829590		.0264634

Use $a = .0265$

$$s^2 = \frac{1}{n' - r}\left[S(R^2) - al_1 - bl_2 - cl_3\right] = .0000350$$

$$s^2 = \frac{1}{12}\left[.0064114 - .0071451 + .0011540\right] = \frac{1}{12}\left[.0004203\right]$$

$$s_a^2 = s^2c_{11} = (.0000350)(.1371471) = .0000048$$

$$s_b^2 = s^2c_{22} = (.0000350)(1.2183150) = .0000426$$

$$s_a = .0022 \qquad t_a = a \div s_a = 12.03$$

$$s_b = .0065 \qquad t_b = b \div s_b = 4.75$$

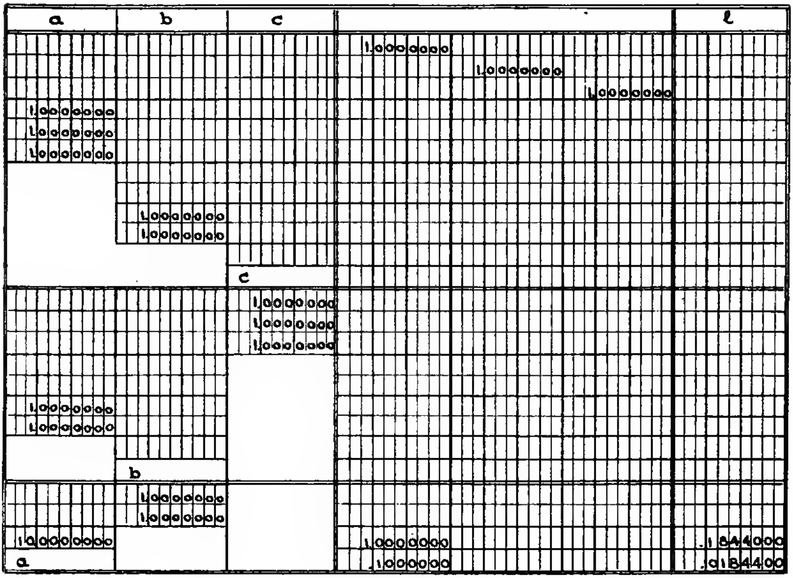
$$d = \sqrt{aa'} = \sqrt{(.0265)(.1190)} = \sqrt{.0032} = .0565$$

TABLE VI-B

Analysis of Retirements—Solution of Normal Equations

Function: $R = a$

Range: 19–28 $\qquad n' = 10$



Use $a = .0184$

$$s^2 = \frac{1}{n' - r}\left[S(R^2) - al_1 - bl_2 - cl_3\right] = .0000011$$

$$s^2 = \frac{1}{9}\left[.0034100 - .0034003\right] = \frac{1}{9}\left[.0000097\right]$$

$$s_a^2 = s^2 c_{11} = (.0000011)(.1000000) = .0000001$$

$$s_a = .0003 \qquad t_a = a \div s_a = 61.47$$

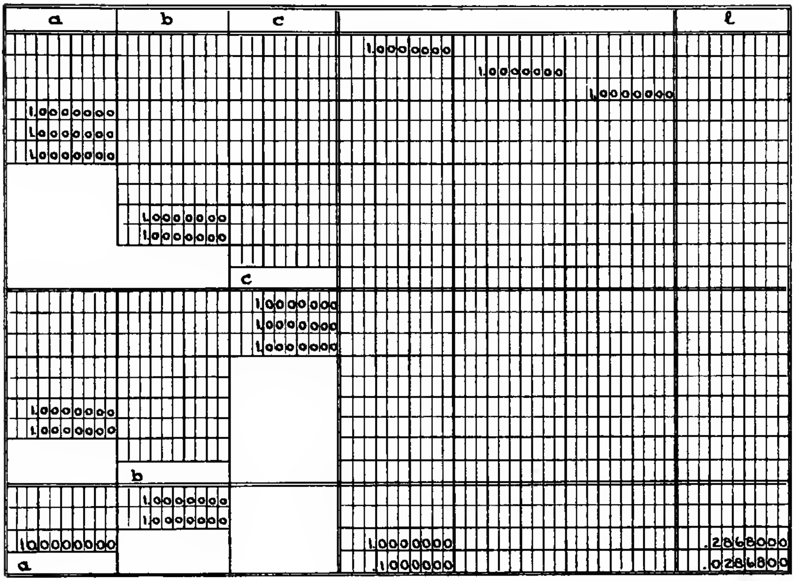
$$d = \sqrt{aa'} = \sqrt{(.0184)(.1864)} = \sqrt{.0034} = .0583$$

TABLE VI-C

Analysis of Retirements—Solution of Normal Equations

Function: R = a

Range: 26–35 $n' = 10$



Use a = .0287

$$s^2 = \frac{1}{n' - r}\left[S(R^2) - al_1 - bl_2 - cl_3\right] = .0000652$$

$$s^2 = \frac{1}{9}\left[.0088118 - .0082254\right] = \frac{1}{9}\left[.0005864\right]$$

$$s_a^2 = s^2 c_{11} = (.0000652)(.1000000) = .0000065$$

$$s_a = .0025 \qquad t_a = a \div s_a = 11.47$$

$$d = \sqrt{aa'} = \sqrt{(.0287)(.1830)} = \sqrt{.0053} = .0728$$

The solution of the normal equations and the evaluation of the constants a' and a is shown in Tables V-A, B and C, and VI-A, B and C, respectively. In the form illustrated, functions containing from one to three constants may be set up and solved. The form provides for the computation of the cofactor, c_{11}, necessary for the estimates of standard errors of the coefficients in the equation used.

In this form a ready check of the calculations is available by a comparison of cofactors and by substitution of the coefficients in the original equations.

The symbols used are as follows:

s^2 is the variance of the function

$s_{n'}^2$ is the variance of the coefficient a' and similarly for the other coefficients

c_{11} is the cofactor of a' in the determinant developed in the fourth, fifth and sixth columns of illustrative Table V

t_n is the ratio of the computed value of a' to the estimated standard error of a' when $t_{n'}$ is found to exceed 2, the value of a' is considered significantly different from zero.

The values of d computed at the close of Table VI-A, B and C, are taken as the best estimates of the reciprocal of average service life at these various points. Under the straight-line theory of depreciation, d equals the annual depreciation rate uncorrected for salvage. The concurrent salvage studies are used to compute the net depreciation rates which are employed to develop Table VII.

In the example used to illustrate the asymptotic method, it would be difficult, if not impossible, to obtain results by the proper application of other methods. There are no readily available records from which a mortality study* can be developed. A good description of the actuarial approach to the depreciation problem is given by Robley Winfrey.† The so-called "turnover" methods may be applied on the basis of additions and retirements from fixed capital, but here the basic theory is at variance with the facts. The method developed by L. R. Nash‡ is an excellent example of the "turnover" method.

Of course, it is sometimes possible to develop some data as to age distribution of dollars retired and the dollars surviving which are needed to construct mortality curves. However, this entails a great expenditure

* Edwin B. Kurtz, *Life Expectancy of Physical Property*, Ronald Press, 1930.

† "Statistical Analyses of Industrial Property Retirements," *op. cit.*

‡ L. R. Nash, "Public Utility Depreciation Accounting," *The Journal of Land and Public Utility Economics*, October, 1926, Vol. II, No. 4.

TABLE VII

Application of Annual Depreciation Rate and Development of Built-up Reserve

Year (1)	Average Balance (2)	(a) Per cent Net Annual Depreciation (3)	Amount Net Annual Depreciation (4)	Net Retirement Loss During the Year (5)	Balance in Reserve at Beginning of the Year (6)	Per cent Reserve at Beginning of the Year (7)
1900, Aug. 1	$ 363,972.64	4.00	$ 6,066.21	$	$	
1901, Jan. 1	419,297.90	4.00	16,771.92	2,341.11	6,066.21	1.61
1902	489,362.46	4.00	19,574.50	1,405.72	20,497.02	4.43
1903	522,044.76	4.00	20,881.79	11,046.81	38,665.80	7.49
1904	539,239.22	4.00	21,569.57	421.29	48,500.78	9.18
1905	548,380.16	4.00	21,935.21	41,566.34	69,649.06	12.66
1906	566,128.68	4.00	22,645.15	19,057.78	50,017.93	9.15
1907	602,085.28	4.00	24,083.41	24,129.97	53,605.30	9.15
1908	632,467.78	4.00	25,298.71	36,492.17	53,558.74	8.66
1909	676,006.23	4.00	27,040.25	10,572.89	42,365.28	6.55
1910	758,321.98	4.00	30,332.88	6,239.57	58,832.64	8.34
1911	884,073.96	4.00	35,362.96	20,459.53	82,925.95	10.22
1912	1,034,942.82	4.00	41,397.71	16,667.60	97,829.38	10.22
1913	1,229,086.90	4.00	49,163.48	19,396.19	122,559.49	11.01
1914	1,437,077.88	4.00	57,483.12	46,224.64	152,326.78	11.32
1915	1,766,538.09	5.14	90,800.06	15,379.74	163,585.26	10.70
1916	2,118,669.39	5.14	108,899.61	66,766.12	239,005.58	11.93
1917	2,378,735.95	5.14	122,267.03	17,419.85	281,139.07	12.59
1918	2,620,162.50	5.14	134,676.35	5,231.14	385,986.25	15.29
1919	2,962,578.38	5.14	152,276.53	14,903.78	515,431.46	18.98
1920	3,716,220.39	5.14	191,013.73	17,424.14	652,804.21	20.34
1921	4,663,174.54	5.14	239,687.17	69,285.31	826,393.80	19.57
1922	5,589,586.67	5.14	287,304.75	57,746.23	996,795.66	19.53
1923	6,679,174.16	5.14	343,309.55	97,692.01	1,226,354.18	20.18
1924	7,945,928.08	5.14	408,420.70	113,362.83	1,471,971.72	20.21
1925	9,536,085.04	5.14	490,154.77	116,142.42	1,767,029.59	20.52
1926	11,589,035.95	5.14	595,676.45	187,734.52	2,141,041.94	20.46
1927	14,159,310.57	5.14	727,788.56	239,839.92	2,548,983.87	20.05
1928	17,075,784.18	5.14	877,695.31	461,250.72	3,036,932.51	19.46
1929	20,394,294.84	4.66	950,374.14	766,794.98	3,453,377.10	18.62
1930	23,453,523.28	4.66	1,092,934.18	1,078,934.79	3,636,956.26	16.35
1931	25,514,390.14	4.66	1,188,970.58	1,056,918.84	3,650,955.65	14.80
1932	27,827,899.72	4.66	1,296,780.13	848,656.42	3,783,007.39	14.35
1933	29,739,780.42	4.66	1,385,873.77	1,187,813.37	4,231,131.10	14.44
1934	30,486,509.83	4.66	1,420,671.36	1,163,028.26	4,429,191.50	14.67
1935	31,949,484.00	4.66	1,488,845.95	1,278,795.13	4,686,834.60	15.22
1936	34,152,762.86	5.68	1,939,876.93	1,115,517.47	4,896,885.42	14.79
1937	36,140,864.74	5.68	2,052,801.12	692,838.37	5,721,244.88	16.26
1938					7,081,207.63	19.09

(a) 1900–1914 5.00* x .80* Depreciable
1915–1928 5.65 x .91 "
1929–1935 5.83 x .80 "
1936 7.28 x .78 "

* Est.

of time and money in most instances because of the mass of detailed data which must be analyzed to develop the basic information required. Even in such instances where this is possible, the basic data are often unreliable and the analysis must of necessity be limited to a few years because of the incomplete data and the cost. A study encompassing a few years' data results in a so-called "band" which may be used to de-

velop retirement ratios at various ages. These data form the basis of the actuarial study. The mortality curve for a particular property may be developed by analogy, that is by finding a "type" curve which seems to fit the observed retirement ratios; or by fitting a curve by the method of Least Squares; or freehand. The objections to the freehand method are simply the fact that the method is subjective and that it is not based on a rigid mathematical foundation. The method of analogy is subject to criticism because it offers no direct relationship between the observed data and the "type" curve and is denounced by both the courts* and the companies in their determination of "fair value" for rate-making purposes. The "type" curves may be developed on the basis of studies of property which is outmoded, obsolete, and often substantially different from the specific property under consideration. Fitting the observed ratios by the method of Least Squares is not subject to these criticisms. However, it is usually impossible and always impractical to reconstruct retirement data for a property's entire history. This leads to a determination of annual depreciation and of accrued depreciation on the basis of the fitted or theoretical mortality curve. The method of determining accrued depreciation on the mortality curve basis is relatively simple. The mortality curve is used to find the life expectancy at various ages. The life expectancy of the property in question, in years, based on its age, is multiplied by the per cent annual depreciation rate to find the per cent of cost new which it is expected will accrue in the future. The difference between one hundred per cent and the sum of these future accruals is taken as the per cent accrued depreciation. It is at this point that the strongest criticisms of the mortality curve method of determining accrued depreciation may be made. The "band" may be too narrow to be representative of the property in the long run. The life expectancy is a direct function of the mortality curve and, therefore, the shape of the theoretical mortality curve selected plays a great part in the determination of accrued depreciation. The theoretical mortality curve is subject to errors of interpolation and extrapolation associated with curve fittings. This means that the actual mortality curve never follows the theoretical curve exactly. Thus, even in the case where the predicted average service life is identically equal to the actual average service life, the theoretical life expectancy need not coincide with the actual future service life at any age. In actual experience, the average service life of groups of units put into operation over a long period of years varies due

* Re Michigan Federated Utilities, P.U.R. 1930D, 506. (U.S.C.C.A.) Arkansas Louisiana Gas Co. v. Texarkana (1938) 24 P.U.R. (N.S.) 267, 96 F (2d) 179.

to changing conditions as well as chance variations. This means that the reserve for depreciation, which is built up in accordance with accounting practice by the application of a uniform annual depreciation rate, will not agree with the theoretical accrued depreciation or the theoretical reserve requirement at all times. In most cases in which accrued depreciation must be determined for valuation or tax purposes, it is desirable to correct for past management errors which have failed to build up a "proper" reserve on the books of account. The management may be willing to admit that it must make adjustments to the reserve for depreciation if such reserves are to be representative for accrued depreciation. It cannot be denied that a company which has made "adequate and proper" provision for annual depreciation under the straight-line theory, where "adequate and proper" is taken in the light of the best available methods of determining annual depreciation, will have an "adequate and proper" reserve for depreciation. This reserve may be called the "built-up reserve." On the other hand, there is no way of proving at any point in time that the future accruals to the reserve will equal the estimates on the basis of theoretical remaining life expectancy. Furthermore, the built-up reserve is developed in complete accord with the Federal Income Tax Statutes* which require that depreciation be charged annually on the basis of facts known at the time such charges for depreciation are determined. Therefore, the reserve based on the theoretical life expectancy may not be "adequate and proper."

The asymptotic method makes it possible to determine the "built-up" reserve when it is impossible to do so on the basis of actuarial studies. In addition, the asymptotic method does not require the assumptions as to a particular shape mortality curve and as to a uniform rate of growth, nor does it impose the condition that two turnover cycles have been experienced all of which are prerequisites for the application of the "turnover" methods. For example, the "Nash"† method assumed the square type survivor curve.

It must be pointed out that the use of the results obtained by the asymptotic method is subject to the limitations of any mathematical depreciation study. The results must be checked with engineers and others who are familiar with the physical property as to the reasonableness of the results as to the amounts of accrued depreciation and annual depreciation in the light of their anticipation of future events. The annual

* See Sec. 23L, 113(b), 114(a), Revenue Act of 1938. Article 23(1)–5. Regulations 101. Treasury Bulletin F, January, 1931. T.D. 4422 and Related Bureau Rulings.

† L. R. Nash, *op. cit.*

depreciation rate developed by the asymptotic method is merely a basis of estimate and is subject to adjustments under special conditions which cannot be taken into account by a strictly mathematical computation based on past experience.

In closing, it may be of interest to illustrate the reasonableness of the asymptotic method by its application to human mortality data. The death rates and birth rates for the ten-year period 1919 to 1928, inclusive, in the United States* were fitted by the asymptotic method. The average life indicated was 61 years. The result agrees perfectly with the weighted average life expectancy at birth of males and females in the United States, which is 61 years, based on the 1929–1931 experience table.

* *World Almanac,* 1939, pp. 323–325.

A DISCUSSION OF "STRAIGHT LINE DEPRECIATION"

A DISCUSSION OF "STRAIGHT LINE DEPRECIATION"

"Depreciation" is commonly defined as "loss in value." This definition is vague in that the terms used have almost as many shades of meaning as there are students of the subject of depreciation. There are many possible definitions of "value" ranging from "prudent investment" through "original cost" or "reproduction cost new or less accrued depreciation" (the latter an example of circular reasoning so far as the definition of depreciation is concerned) to "salvage less cost of removal"; the "loss" at any point in time may be taken as a function of time itself, a function of use, or a function of time and interest (itself a variable), a function of physical or economic forces and effects, and in many other ways.

It is obvious, therefore, that a more rigid definition of "depreciation" is required before any intelligible discussion of methods for its determination can take place. Once such a definition has been postulated, however, it is possible to analyze any proposed method, and show to what extent it succeeds, and how it falls short, in developing results which are consistent with the definition upon which it is predicated.

"Straight line" depreciation postulates a definition of depreciation in which "value" is measured preferably by "original cost less accrued depreciation" (although the percentages of "lost value" so defined can be, and commonly are, applied to estimates of cost of reproduction new to provide corresponding estimates of "cost of reproduction new less accrued depreciation"); and in which "loss in value" occurs as a linear function of time.

In public utility valuation and rate making, this definition has commonly been rejected by the courts in favor of what has been referred to as "actual depreciation," which postulates a definition in which "value" is a complex involving consideration of "original cost," "reproduction cost less accrued depreciation" and other "measures of values"; and in which "loss in value" is described as "a fact" to be determined by engineers at a point in time as a function of various physical, economic and other forces customarily listed as wear and tear, physical deterioration, the action of the elements, obsolescence (or "obsoleteness"), progress in the arts, inadequacy, change in demand, the requirement of public authority, etc.

Nevertheless, due in part to the failure of engineers to develop objective standards for the measurement of the effects of these causes of "loss in value," and thus to provide convincing determinations of "actual depreciation," "straight line depreciation" has continued to be accepted, by default, in some court and commission cases, and still has its ardent advocates, even where it is not so accepted.

Under these circumstances, it is well worth while for engineers to study the methods advanced for determining "straight line depreciation"; and to consider the extent to which they succeed, or fall short, in developing results which are consistent with the definition upon which they are predicated.

Under this definition, as previously stated, "straight line depreciation" would require the spreading of the original cost of property (less net salvage) in equal annual installments, over its service life. If a rigid adherence to this definition were required, its correct application would obviously be impossible, since a precise knowledge of the exact life of each unit of property, and of the net salvage recovered on its retirement, would be necessary at the time of its acquisition. In practice, life estimates which are varied from time to time in the light of experience are used as the basis of annual charges for depreciation, and estimates of "loss in value" at a point in time are based upon "lives" determined at the same point in time, with retrospective utilization of previously recorded facts and with knowledge of the age of the units under consideration.

Of course, the consistency of the results of any straight line method with the definition of "straight line depreciation" depends on the accuracy of the estimates of service lives. The difficulty of proving the accuracy of such estimates is one of the greatest objections to the use of such methods. If one hundred per cent accuracy in forecasting lives were imposed, all such methods would have to be abandoned at the start. However, no method of estimating depreciation in accordance with any definition can be made to adhere rigidly to its own requirements and criticisms of straight line methods ought not to be limited to their lack of perfection.

Basically, straight line methods can be divided into two principal groups. First, the annual depreciation and the accrued depreciation for a group of units of property may be based on the sum of individual estimates for each unit in the group. This may be called the individual unit method. The shortcomings of this method are easily pointed out, and have practically led to its general abandonment. If estimates of

service life are to be made for each unit of property, such estimates can seldom rise above the level of mere guesses. Experience offers no help in the matter of estimating lives for single units because, in the nature of things such experience cannot include knowledge of the life of the individual unit under consideration and still in service. Even if it be assumed that fairly accurate forecasts of *average* service life may be made for large groups of units, no statistically reliable estimate can be made for a single unit.

Second, the annual depreciation and the accrued depreciation for a group of units of property may be based upon their average age and estimated average service life. This is called the group method and is the one in common use in connection with public utility depreciation valuation and accounting.

The group method, in turn, is subject to a number of different variations in its application to the determination of the amounts of annual and accrued depreciation. These variations may be grouped in two principal classes. One includes the so-called "actuarial" methods, which are based on the principles which have been established by insurance companies in connection with human mortality. The other group includes the "turnover" methods, which are based upon analyses of dollar additions and retirements.

Basically, the actuarial methods require a detailed factual background reflecting the number of units or dollars retired each year as well as the installation dates thereof, and the number of units or dollars out of which such retirements were made. From these data mortality or survivor curves are developed, which are smoothed and extended, either freehand or mathematically to obtain a theoretical survivor curve applicable to the property analyzed. The area under this theoretical survivor curve is the basis of the indicated average service life used, and its reciprocal is the annual depreciation rate. The main objection to the use of this method is the usual lack of the basic data required, and the expense of their compilation in proper form even when available. Another important limitation is that the use of units in the analysis does not apply to the dollars to be depreciated when changes in price level occur. If dollars are used instead of units of property in developing survivor curves, such curves are often distorted due to dollars based on varying price levels associated with the retired and surviving units of property. Furthermore, basic data as to installation dates are often not factual but estimated, because of lack of information, thus making the actuarial results of fictitious accuracy.

Granting that the average life estimates obtained by actuarial methods may reasonably be used, there is another series of errors which is often committed in determining accrued depreciation. In order to determine accrued depreciation the age of the surviving units or dollars must be known. It is to be noted that the age is found from the same basic data from which the survivor curve is developed and is subject to the same limitations as to accuracy. The ratio of the average age of the surviving units of the group to the average life determined from the theoretical survivor curve, is sometimes taken as the ratio of accrued depreciation. Such determinations, however, are necessarily inconsistent with the definition of "straight line depreciation" if some units in the group have been retired before the expiration of the average service life. To correct for this error, the ratio of the average age to the probable average life of the surviving units is sometimes taken as the measure of accrued depreciation. This ratio still fails necessarily to accord with the facts, because the reserve representing the accrued depreciation of the surviving units would be depleted below this ratio by debits for the full cost of retirements made before the expiration of average service life. An attempt to correct all these faults has been made by determining the accrued depreciation ratio by deducting from unity the estimated future accruals to the reserve, which are computed as the product of the annual rate times the probable average remaining life (probable average life minus average age) of the property surviving. There is no doubt that in theory the last would be the correct solution if every addition and retirement in the past had occurred, and if every addition and retirement in the future would occur, exactly in accordance with the requirements of the theoretical, smoothed survivor curves. In practice, however, variable life estimates are used and actual additions and retirements do not occur along the path indicated by the smoothed survivor curve. Of course, price level changes are another disturbing feature to be considered in this connection. Therefore, it is only by coincidence that the accrued depreciation ratio at any time as determined above will agree with the reserve for accrued depreciation which would result from the application of consistent annual depreciation rates, and the recording of additions and retirements when they actually occur.

The "turnover" methods are based on analyses of dollar data as to retirements and additions. The basic data are more generally available, and are usually more reliable than those required for the actuarial methods, since the dollar turnover forms the basis of the most elementary accounting records. Since depreciation accounting deals with the ac-

crual of dollars, the "turnover" methods are not disturbed by changing price levels. An increase in price level applicable to additions has the same effect as a corresponding increase in the number of units added with a fixed price level. The main limitations of the "turnover" methods are the distortion of estimated lives which results from growth of the group, and the necessity for long periods of experience before these methods can be used properly.

These limitations have been overcome to a considerable extent by the development of the so-called "asymptotic" method of determining annual and accrued depreciation. By the use of this adaptation of the "turnover" method, annual rates may be determined without actuarial data, and without the necessity of assuming a particular shape of mortality curve or a uniform rate of growth. Furthermore, it is not subject to the requirement of two complete turnover cycles, as required for other "turnover" methods, and it permits the accrued depreciation to be determined by means of a "built-up" reserve. The "built-up" reserve is developed in accordance with accounting procedure, by crediting the reserve with annual depreciation on the cumulative balance as it is actually built up, and debiting it with net retirement losses as they actually occur.

While the "asymptotic" method, based on the geometric mean of the limits of the retirement ratio and the additions ratio, as computed from fitted polynomial functions, itself yields results inconsistent with the facts for extreme cases of abnormal average lives and growth ratios, analysis demonstrates that within the limits ordinarily encountered in studies of utility property, it yields a close approximation to the theoretically correct annual depreciation. In such cases the "built-up" reserve may reasonably be taken as a measure of the accrued depreciation. Furthermore, when the type of survivor curve applicable to the property is known or assumed, errors due to extreme cases of average life and growth ratio can be corrected within reasonable limits by resort to tables or curves prepared for the purpose.

The availability of improved actuarial methods and the "asymptotic" variation of the "turnover" method of computing "straight line depreciation," however, should not obscure the fact, pointed out at the beginning, that all such methods are based upon an erroneous assumption that "loss in value" proceeds in accordance with a linear function of time, instead of a function of the physical, economic and other causes of "actual depreciation." Instead of affording an argument against engineering studies of "actual depreciation," they should spur engineers to

the development of more complete objective standards of measurement of physical and economic change, as they may reasonably be considered to cause "loss in value"; and to the development of practical and acceptable methods of determining "annual depreciation" in correspondence with retirement losses and changes in accrued or unrealized "actual depreciation."

PART TWO

ESTIMATED "ACTUAL" DEPRECIATION OF PUBLIC UTILITY PROPERTIES

BY

FRANKLIN JEFFERSON LEERBURGER

CONTENTS

PREFACE

The problems of the public utility industry relating to engineering designs, costs of construction, finance, and special accounting matters have interested me professionally for many years, first as Principal Assistant Engineer in the office of Mr. Maurice R. Scharff, Consulting Engineer, lately as Principal Valuation Engineer of the Public Service Commission of the State of New York.

In the conduct of investigations for such public utilities as the Bronx Gas and Electric Company, the Westchester Lighting Company, the Rockland Light and Power Company, the Duquesne Light Company, and others, it was necessary to consider such matters as rate base, cost of reproduction new, together with a host of other esoteric terms which form the stock-in-trade of the engineers, accountants and lawyers engaged in regulatory proceedings. To the uninitiated it appears that because these questions are to be litigated, the fundamental concepts and terms would be clear and universally understood. For this reason it was disturbing to learn that the terms, *value* and *depreciation*, could have so many different meanings.

Studies of United States Supreme Court decisions in recent rate base determinations lead to the conclusion that the Court preferred estimates of actual depreciation to mathematical formulations,* and for this reason, at least, one prominent engineer has repeatedly testified that depreciation is a social and legal concept and not a physical or economic one.

Professor Bonbright devotes an entire chapter of his *Valuation of Property*[1] to the "significance of the statement that 'Value is a Word of Many Meanings.'" Therefore, what purpose is served in making meticulous field inventories, detailed and exhaustive estimates of costs of reproduction, or of accrued and annual depreciation? All are alleged to have some relation to "value," but Professor Bonbright, quoting Humpty-Dumpty, says: "When I use a word it means what I choose it to mean—neither more nor less."

It is the purpose here to discover, if possible, an underlying principle, something more substantial than "social and legal" concepts. No such discovery resulted from this effort, but it is evident that perhaps certain questions are but pseudo-questions and that many difficulties which

* Pacific Gas & Electric v. San Francisco, 265 U.S. 403, P.U.R. 1924 D817. McCardle v. Indianapolis Water Co., 272 U.S. 400, P.U.R. 1927 A15.

beset the workers in this field rest on the myths and folklore of our social institutions. A conclusion which may be drawn, although I am still without proof, is that some seeming problems would vanish if more thought were given to the implications of the meaning of meaning as expressed by Ogden and Richards[2] and by Alfred Korzybski[3] and popularized by Stuart Chase.[4]

Volumes have been written on the meaning of value and depreciation. These are very largely devoted to definitions which go on and on until all meaning has been obliterated. Thurman Arnold[5] has said:

> *** This leads to the *** trap which lies in wait for the diagnostician of modern society. That trap consists in a faith in definition. The only purpose of logical definition is to resolve mental conflicts. It is not useful as a descriptive tool because it is ordinarily used as a method of finding out what "things" really are, instead of as a method of conveying thought by the quickest available means. The law, which is above all a method of reconciling conflicting ideals, becomes so heavy with definitions that it is almost unintelligible. Therefore, it becomes a convenient illustration of the traps in which learned people are particularly liable to be caught.
>
> Definition is ordinarily supposed to produce clarity in thinking. It is not generally recognized that the more we define our terms the less descriptive they become and the more difficulty we have in using them. *** Logical definition enters when we are using words which we are sure "ought" to mean something, but none of us can put our finger on just what that meaning is. (Page 179)

It was at the urging of Professor Theodore Baumeister and of Mr. Maurice R. Scharff that I consulted with Professor James C. Bonbright from whom I received the necessary encouragement and stimulation to undertake this study. A part of the task included a reconsideration of the implications of what valuation engineers were doing with the depreciation problem, and to develop a method for determining the magnitude of accrued and annual depreciation which would rest on firmer grounds than on court decisions and legal fiction. The attempt has been made to give the words "value" and "depreciation" meaning through examples and other illustrative means, without resorting to prolix definitions. Taking a hint from Korzybski and Chase an effort has been made to develop a semantic approach by developing a set of standards which, if eventually accepted, may relate depreciation to value.

Fundamental to these studies is the realization that depreciation and value have specific implications and perhaps meanings, only within a particular social and legal framework.

Certain beliefs which furnish some background for the consideration of the implications of the arguments advanced must be set forth. One is that the courts have tried with little success to establish standards of value in regulated monopolies which are analogous to standards of value obtaining in competitive enterprises. Another is that the so-called fair value doctrine has been advanced by the courts for the purpose of approaching as nearly as possible to the competitive ideal. The exploration of the relation of depreciation to such a concept of "fair value" seemed to be a natural consequence of these beliefs.

Some of the questions raised are unanswerable in the present state of our knowledge of society. Nevertheless it is hoped that a method of determining depreciation will be suggested in the following pages and that this method will appeal to those faced with the necessity of dealing with this problem, as being realistic and consistent.

Many friends and colleagues have given me valuable suggestions and I am especially indebted to Professor Bonbright for his provocative discussions and for the time which he gave to me unsparingly. Professor Horace Taylor took unusual pains to try to guide me through the paths of classical economics in its relation to the modern capitalistic system with its elements of monopolistic competition.

Mr. Maurice R. Scharff made available the facilities of his consulting office and contributed much from his wide experience. Mrs. Bernard Kappalman of the Abraham Lincoln High School has made extensive comments and corrections; the errors, however, are my own. Many others have read the earlier drafts and have criticized the various points advanced. I am grateful for the help rendered by Professors Robert T. Livingston and Frederick C. Mills, and for the patient guidance of Mr. Samuel B. Seidel of the New York Bar. Mr. Joseph Jeming constructed many of the mathematical proofs and supplied much of the statistical basis for the study. Others who have read the drafts and criticized these efforts, include Mr. Walter Lyman of the Duquesne Light Company, Mr. John Balet of the Consolidated Edison Company of New York, Incorporated, and Mr. Sidney Newborg of the New York Bar.

CHAPTER I

STATEMENT OF THE NATURE OF THE DEPRECIATION PROBLEM AND ITS BACKGROUND

Professor Glaeser[5b] has said:

> The need of disclosing the cost at which different operations are being carried on is an extremely important objective of accounting control. Management and regulatory authorities are alike interested in this aspect. This means that the accounts must be organized in such a way as to show the cost of major operations. (Page 31)

The recording of all facts in respect of current income and current expenses, as well as the presentation of the history of all past incomes and expenditures is, of course, the primary function of any accounting system. There are other functions or purposes, among which is the presentation of data for comparing different operating and managerial practices and improving them. Uniformity and comparability of accounts are the essential attributes to any comparison.

Accounting should, therefore,* reflect costs without the application of adjustments for the purpose of considering "value" or changes in the price level. A recent writer[6] of accounting technique has, nevertheless, suggested the need and advantage of what he is pleased to call "stabilized accounting," by means of which he hopes to adjust all facts in respect of costs, by factors which would present original costs in terms of a constantly changing price level.

The distinction between current assets, fixed capital and operating expenses has been presented in many texts† and will not be discussed here.

A statement of the financial condition of an enterprise at its inception or expiration, involves no special difficulties in respect of the status of the fixed assets. Intermediate statements, drawn off before the retire-

* If the accounts are frequently adjusted to reflect a changing price level and if the property, as a matter of fact, was installed over a long period of time, then there is no uniformity which is intelligible and the accounts are not comparable in respect of their significance. This constitutes the reason for the thought that accounting should reflect actual costs without such adjustments. If it becomes necessary to determine for valuation purposes what the probable present cost would be, accounts and sub-accounts might be restated in terms of the present price level.

† For an adequate summary see Glaeser, pages 133–145.

ment of fixed assets, require the consideration of depreciation, that is, the consumption or loss in value of the assets during the life of the enterprise, up to the date of the inquiry. This does not mean, however, that the assets lost value or were consumed uniformly in the intervening period. An essential consideration is that the units, in which the loss or consumption are expressed, shall be the same as those in which the original cost was set forth.

Accounting, because it is expected to present factual conditions, should not be expected to disclose value, for the reason that changing price levels, judgments and opinions are involved.*

It is possible to estimate, in valuation studies what a given physical property might have cost to reproduce at any time, other than the time at which it was actually purchased or installed. It is likewise possible to estimate in terms of this hypothetical cost, what the corresponding accrued depreciation is, in terms or units of this hypothetical cost, rather than of the original or actual cost. These are valuation procedures and should not be made parts of the accounting technique.

In determining a rate base for a public utility enterprise, one of the considerations is the actual cost of the used and useful property less the accrued depreciation of that same property. Thus it is seen that the books of accounts must serve the dual purpose of divulging financial status and of setting forth at least two considerations in rate base determination. It is therefore essential that the portion of expenses due to annual depreciation be consistent with the amount of accrued depreciation to be deducted from the cost of the fixed assets in determining a rate base.

The public utility industry has not universally understood the necessity for such consistency. The Federal Communications Commission in its Proposed Report—Telephone Investigation Pursuant to Public Resolution No. 8 of the 74th Congress stated at page 401 that:

> These inconsistencies between company claims for large annual depreciation charges to be included in operating expenses and simultaneous contentions for relatively low amounts of accrued depreciation to be deducted for valuation purposes have given rise to major controversy in a large number

* Value implies many things not involved in a mere recitation of actual costs. If the money for capital additions has been prudently spent, then we may assume that cost and value are synonymous at the time of installation but with a changing price level this is no longer true (except by fortuitous circumstance) and it is also no longer true if changed market conditions would not call forth the same expenditure were the plant to be built at some other time even if the price level had remained the same. Further comments on this point are advanced in the appendix.

of rate cases, notably in the New York Telephone case (36 Fed. (2d) 54) in which the Federal Court held that the depreciation reserve was the best evidence which had been presented to it of the actual depreciation existing in the property, and in the Illinois Bell case (292 U.S. 51) in which the Supreme Court held that differences in the two kinds of depreciation as claimed by the company were irreconcilable.

Earlier in the same report the F.C.C. stated that—

> The pursuit of this argument has resulted in rate case claims by Bell System companies of accrued depreciation for valuation purposes limited usually to from 5 to 10 per cent only of the original cost of the properties, whereas the same companies have accumulated reserves from annual charges to operating expenses on account of the same properties amounting to from 20 to 35 per cent of the plant.

At this point it is necessary to state what the symbol, "value," is to signify throughout the discussion.

Because language is incapable of conveying every idea in its fullest implication, it frequently happens that a particular word will not accurately represent and convey the intended meaning. Examples, gestures and the like may assist in conveying meaning where the mere word fails. I shall, by examples, attempt to round out what I mean to convey by the word *value*. As an introduction, I shall take it to mean, not what an asset is actually worth to a company in view of its being a regulated monopoly, but what an asset would be worth on the fictitious assumption that the utility business operates under conditions of competitive price determination.*

The determination of a rate base by a court or commission does not thereby, necessarily, fix value. If, for example, a business is subjected to the rigors of competition it may be that the rate of return and the rate base are such as to call for a permitted return greater than the actual one realized. Value is, then, less than the rate base. For example, a railroad may be unable to earn prime costs and yet have a positive rate base. Its value, in accordance with the introductory comments, is less than zero; it is worth nothing as a producer of net returns to the owners and yet it has an associated rate base.

On the other hand, a public utility which is operating in a market where its earnings, if unfettered by regulation, would rise far above the

* An alternative view is that value of a depreciating asset is what the asset would be worth *if* the plant were owned by the rate payers themselves and *if* it were built with all of the knowledge available at the time the determination of the rate base was being made.

permitted return illustrates the limit to which value itself may rise. If regulation of such a utility does not permit the earnings to rise to this ceiling, then regulation sets the limit on value proportioned to the permitted earning, in fact regulation has determined the value.

It is not urged that the rate base and rate of return are determined entirely by reference to imaginary standards set up by a fictitious competitive situation. The view that regulation is generally designed to simulate the desirable effects of competition is supported by such writers as Kirkland,[7] Hacker[8] and VanMetre.[9]

Speaking of the passage of the Interstate Commerce Act and the Sherman Anti-Trust Act Kirkland says:

> Another reason for the general acceptance of the legislation was its conservative character. It did not break sharply with the American policy of individualism under which individual capitalists were allowed to go their own way and private property was not to be tampered with. Implicit in this theory of individualism or laissez-faire was the comfortable belief that competition would bring benefits to the mass of people, and that the individual's quest for gain would result in some community advantage. (Page 623)

VanMetre, who has spent a lifetime considering transportation rates and regulation, thinks that:

> When the government begins to establish control over private business enterprise, the most important and often the only end it has in view is to secure prices that are just and reasonable for the goods and services which the public buys. Two methods may be employed to achieve this end. The government actually fixes prices; or it endeavors to preserve competition, with the hope that the competitive struggle among rival producers will keep prices at a reasonable level. *** In its endeavor to secure fair railroad rates the federal government has tried both methods. That is, it has tried to preserve competition in the transportation industry, and it has also resorted to price fixing.

What a just and reasonable rate may be is itself a high order abstraction difficult to explain. A most interesting presentation of the historical development of regulation is given by Hacker[8] in discussing the Interstate Commerce Act and he too stresses the philosophy underlying regulation, namely the preservation of the competitive standard.

The evolution of the procedures for regulating prices has led to the considerations of "fair value" and "fair rate of return." These are terms conveying little meaning. The original cost of a property less the ac-

crued depreciation might be considered as a rate base. The application of a rate to such a base might then yield a return which in part would have been determined by competitive considerations. For the accrued depreciation would have included considerations of the possible effects of competition and the rate of return would have been determined with reference, not only to the need for attracting capital on the one hand, but also for recognizing possible competitive influences on the other. Whether cost of reproduction, original cost or some adjusted weighted average of these costs be used as an initial figure, we should, nevertheless, turn to assumed competitive conditions for guidance in determining depreciation. For convenience in presenting these ideas we may assume, for present purposes, that book costs and costs of reproduction new are identical and that there are no other factors, except accrued depreciation, which would be considered in determining a rate base. In other words, the complications of a changing price level are eliminated from consideration. The purpose of this explanation is to make clear that in the subsequent discussion, a fair rate base is assumed to be original cost minus accrued depreciation. It is expected that this approach will avoid cumbersome reiteration of what is meant by a fair initial point and will identify it, for reference purposes, with original or book costs. In practice, rate bases are frequently fixed by some mental weighting of actual cost, cost of reproduction new and trended book cost from each of which appropriate deductions for accrued depreciation are made. The process of weighting and the considerations to be given to the numerous elements bearing on the finding of an undepreciated starting point are subjects for long and detailed examinations which will not be undertaken here. The difficulties will be circumvented by assuming that book cost, trended book cost and cost of reproduction new are identical. A further simplification will also be introduced by assuming that at the time of retirement there is no net salvage. That is, the difference between salvage and costs of removal equals zero.

There is no business, other than a public utility, which sells its products or services at prices determined by a rate of return applied to some rate base. The object is to explore the concept of depreciation as being the loss in value due to specific causes which would effect such loss in a business subjected to competition. It may very well be that such a concept of depreciation is not the only valid one, but the purpose here is to explore this one and not other possible ones.

APPENDIX

The assumption made in this paper is that there is no change in price level and that all of the *judgment* with respect to loss in value is reflected in the estimate of actual accrued depreciation.

Even the original cost principle of rate making does not completely abandon the competitive price criterion of rate making. Instead, it makes just *one* modification; namely, that it declines to recognize changing price levels as an occasion for a change in rate base. Hence the same *percentage* of depreciation must be deducted from original cost as would have to be deducted from reproduction cost new if this latter standard were adopted. At this time it is not clear that the same concept of depreciation is applicable both to original cost and to reproduction cost rate making. But the implications of the following example are worthy of further investigation and testing.

The actual cost when new of a property was $1,000.00 and presumably this represented the value of the property at the time of construction. After the passage of some time an examination and study reveal the facts that the identical property would be built today if the property were not actually in existence. The price level, however, has changed and the cost of reproduction new is $1,500.00. The present value presumably is $1,500.00 in terms of present price level and $1,000.00 in terms of original price level.

Now if the property has lost "value" to the extent of 30 per cent due to obsoleteness or any other cause or combination of causes of depreciation then the "value" of the property in terms of original cost is $700.00 and in terms of present price level is $1,050.00. For our purposes, therefore, it is unnecessary to distinguish between the two price level "costs."

In terms of cost of reproduction new the loss in value was $450.00 and in terms of original cost the loss in value was $300.00, the percentage in either case being the same. Value for rate making purposes must be determined upon consideration of both $700.00 and $1,050.00. This paper does not concern itself with the weight to be given to each of these figures nor does it concern itself with the other imponderable elements to be stirred and mixed into a "brew" called "fair value."

This, of course, begs the question of what "value really is" by the assumption that whatever it may be the same *per cent* of accrued depreciation should be deducted from each of the factors, be they cost of reproduction new, original cost, prudent investment, etc. A change in price level so far as it would influence the costs to reproduce an identical property, if this were undertaken, must be ruled out as a force influencing the appreciation or depreciation of the value of the property.

It certainly would not influence value in this regard in a competitive situation and would only influence value *if* the regulatory authorities changed the rate base with changing price level *and* the revenues changed only propor-

tionately. For if the price level rose and the rate base rose as a consequence and the permitted return were higher than the consumers would underwrite, then "value" would not have increased with increased price level. Conversely if a falling price level called forth a proposed reduction in revenue by a lowering of rates and if the consumers greatly increased their consumption as a consequence of the "bargains" to be had, then "value" would rise despite a fall in price level. Perhaps the most that should be said, is that whatever one might name the effect of changing price level or the rate base or cost of reproduction new, etc. it is not the same concept or thing as depreciation or appreciation. It may be equal at some particular time to a corresponding amount of either appreciation or depreciation but equality in dollar magnitude is accidental or fortuitous.

CHAPTER II

EXISTING METHODS OF TREATING DEPRECIATION AND A DISCUSSION OF A TEST FOR CONSISTENCY OF ANNUAL AND ACCRUED DEPRECIATION

The standard methods of accounting for depreciation in competitive businesses in the United States have been designed to amortize the original book costs of property. The particular method selected usually depends on some managerial or financial policy of allocating the burden of expenses. Thus, under the sinking fund method the reserves for retirement increase at a changing rate, while under a rigid application of the straight line method there are equal annual credits to the reserve. The retirement expense method, on the other hand, places the entire burden at the end of life. In using the straight line or the sinking fund method, lives must be estimated although these predictions need not necessarily be correct. The aim of cost accounting is to allocate total costs to total production so as to make each unit or quantity of product bear a proportionate share. This makes it desirable to predict lives as accurately as possible.

Turning from competitive to regulated enterprises, where the return on capital is computed by reference to rate base and not by the excess of marginal prices over marginal costs, depreciation has a new significance. This can best be studied by comparing several of the better known methods of depreciation accounting.

Before comparing these methods it may prove helpful to give a number of short and general definitions describing the sense in which certain terms will be used. It will, of course, be necessary to supplement and to amplify these as the discussion proceeds. The term depreciation of necessity is given a fuller treatment.

Retirement Expense Accounting is that method by which the actual original cost is written off upon retirement.

*Installation Expense Accounting** is only a theoretical method diametrically opposite to the retirement expense method. At the time of installation a reserve would be set up equal to original cost of installation.

Straight Line Accounting is that method by which the life of prop-

* There is no such method in practice.

erty is estimated and corresponding annual increments are uniformly credited to the reserve.

Sinking Fund Accounting is like straight line accounting, except that the accruals to the reserve are adjusted for the compound interest to which each is subject.

Depreciation signifies the effects of certain facts, some of which are physical and some of which are economic. The two categories are not mutually exclusive, but may be useful in analysis. The physical facts to which reference is made, include deterioration, wear, tear and decay. The economic facts include obsoleteness, change in use, requirements of public authority and growth or decay in a particular demand. Without laboring the point, it should be noted that obsolescence in a machine is usually due to physical changes in some other machine. The depreciation of physical property is associated with the loss or diminution of certain attributes or characteristics such as strength, weight, color, shape, capacity to perform a service or to supply a need. For certain purposes, engineers have devised appropriate units for use in comparing and measuring the effects of these physical facts, whereas the economists have been more or less successful with respect to the units for measuring economic facts. It is evident that for accounting purposes, all these facts must be expressed in units which are generally understood and commonly accepted. Moreover, it is essential that monetary units be adopted.

In the realm of physical observations, the use of conventional units is based on an agreement to accept certain arbitrary standards for the purpose of recording data and of communicating findings or repeating experiments. The problem of expressing depreciation in terms of monetary units will be taken up subsequently.

It is necessary at this point to introduce a standard for the determination of consistency in the treatment of accrued and annual depreciation. In a later chapter several tables are included, the purpose of which is to demonstrate the proposition that if the return on capital and the credits to the depreciation reserve are treated with due regard to each other, then regardless of what method of depreciation accounting is used, a consistent pattern will result. What that due consideration and consistent pattern are, may be stated as follows: if the annual return on the difference between book capital and accrued depreciation is added to the corresponding annual accrual of depreciation and this sum is multiplied by appropriate "present worth" factors, and if these products are computed annually and summed up over a term of years equal to the life of the property in question, then the result will be an amount of

money equal to the original investment. An appropriate present worth factor is a multiplier which expresses the reverse of a corresponding compound interest factor. Thus, assuming a rate of return of 7 per cent, the present worth of one dollar to be returned at the end of one year is \$0.934579 at the beginning of the year. That is, \$0.934579 invested at 7 per cent will, with one year's interest, equal \$1.00. Similarly \$0.873439 invested at 7 per cent will with two years' interest compounded annually equal \$1.00 at the end of two years. Therefore \$0.934579 is the present worth of \$1.00 at 7 per cent return to be received after one year and \$0.873439 is the present worth, at the same rate of return, of \$1.00 to be received at the end of two years. The present worth factor is the multiplier which, when applied to a given payment to be made at some specified future date at some specified compound interest rate, will yield the present worth of that payment. The present worth concept expresses a classical economic preference for a present value as against a future one.

Marshall[10] at an earlier date made some rather trenchant comments on these matters but some of the reasoning might be challenged by certain modern economists. For purposes here the time-discount is a fact in our economic system.

For the purpose of studying the effects of the various methods of depreciation accounting, let us consider a seasoned firm, that is one which is operating under the following conditions:

1. At least one life cycle has been experienced, i.e. all of the property installed during the period of growth has been retired.
2. The capital investment is constant from year to year, i.e. there is no further growth or diminution to be expected, i.e. the additions and retirements are approximately equal from year to year.

Certain necessary assumptions should be stated before proceeding with the analysis:

1. The price level remains unchanged.
2. The life of the business is infinite although particular parts of its property expire in finite periods.

These conditions are all necessary to the study of the business in which one condition at a time may be varied while others remain unchanged.

Under the foregoing circumstances, the application of retirement expense accounting results in the writing off, at time of retirement, of the

original cost of each piece of equipment. The rate base* is reduced by an amount corresponding to such retirements, but, as we are dealing with a seasoned business, additions equal in money expenditure to book cost retirements are being made. As the retirements and additions offset one another on the books, the rate base remains constant. The retirements are charged as operating expenses, and so each year the rate payers pay a sum equal to a return on a constant rate base plus the operating expenses and the cost of the currently retired portion of the property. The latter are really repayments of original investment. Different year groups of rate payers bear their equal burdens of depreciation expense indicating that retirement expense accounting, under the assumed conditions, would impose equal charges for equal services.

Let us examine the conditions which obtain during the period of growth, that is, before the business has become seasoned or stabilized. The rate payers in the first year of operation would pay a return on the total original cost. During the first year there will have been only a small number of unimportant retirements so that the retirement expense borne by these early customers is small. The retirements in the succeeding years would increase from year to year because of the effects of depreciation upon increasing amounts of property, until the inauguration of the stabilized era. The additions also would increase each year until stabilization. The changing ratio of retirements to book balances would place a minimum burden of retirement expense on the first customers and an increasing burden on later customers until seasoning is achieved. This is so because property in general is not retired immediately following installation. In the early years of operations there may be small scattered retirements but the average ratio of retirements to book balances will not manifest itself until we enter the stabilized period.

Similarly, if a seasoned business retired after many years of operations, the rate payers in the last year would have to repay the company for the retirement of all property then in service or else the investors would have to stand the loss. There is a dilemma here; both alternatives fail to equalize the losses. It may be argued that this cause of the retirement of a business may not be depreciation. For the purpose of illustrating certain effects, however, it will be assumed that the cause of the retirement is the decline of the demand for the services of the company.

* We are dealing here with a public utility.

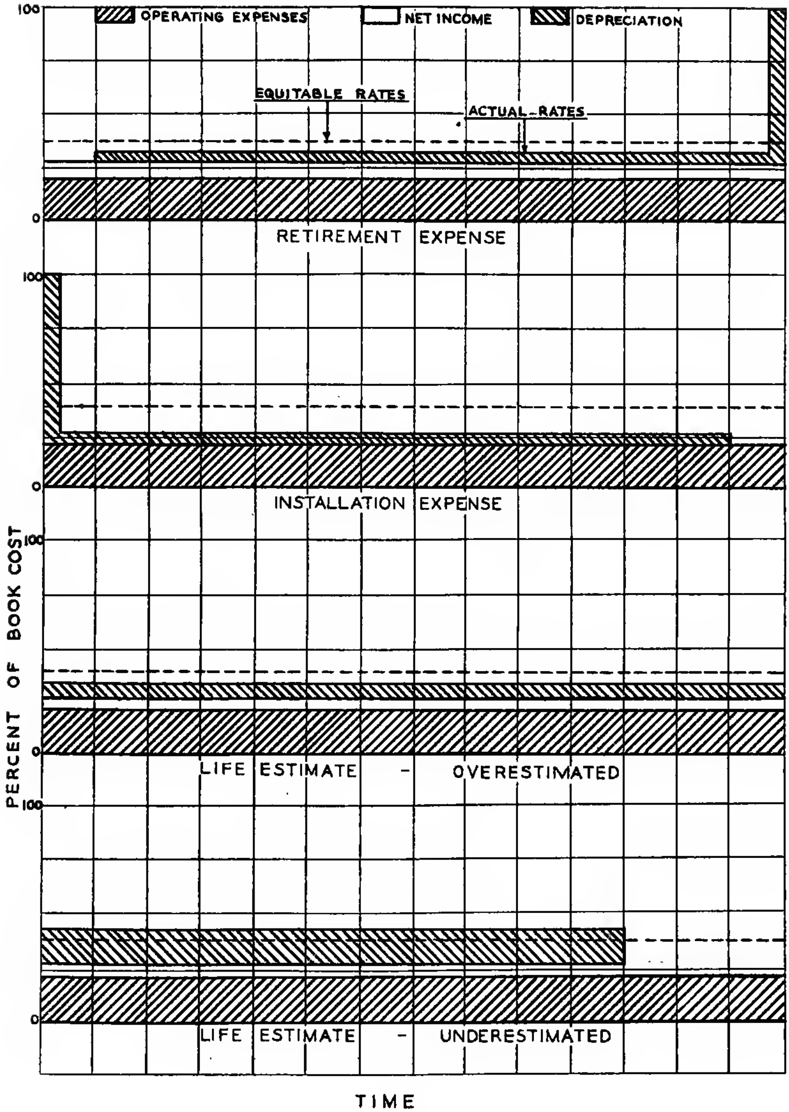
In the first example relating to retirement expense, the apportionment of the loss burden has followed a plan of reasonable equalization between the company and the rate payers as a whole, while ignoring the wholly unequal apportionment between different year groups of rate payers.

The accompanying diagram, among other things, illustrates how reimbursements may be distributed over the life of a business. Some consideration of the shape of the loss distribution or reimbursements leads to the conclusion that equalization of the burden would be effected by increasing it for the earlier rate payers and slightly decreasing it for those in the stable period. Of course, such redistribution could not be made without the benefit of all the experience had throughout the lifetime of the business, and it would therefore be impossible to arrive at an appropriate burden for each year group except in retrospect. Although the question is therefore only of academic interest, the diagram illustrates the relationship of the equalized burden to the unadjusted one.

The installation expense method is analogous to the retirement expense method, excepting that accruals to the reserve are equal to the costs of the additions and not of the retirements. This is not a method used in actual practice, but is inserted here as the antithesis of retirement expense accounting for purposes of contrast. We again should observe the disproportionate burdens in the beginning and ending periods as indicated in the diagram. Equalization requires that the early rate payers be relieved of some of the burden at the expense of the later ones.

These extreme methods of reimbursing the owners for losses set the limits of the problem. All other methods, such as sinking fund and straight line, attempt to redistribute the demonstrated disproportionate burdens.

The next method to be considered is straight line depreciation accounting which requires that an estimate of the actual life of a property be made, but since it is highly improbable that the actual life would be exactly estimated from statistical data, we shall confine our analysis to the results obtained on the one hand from under-estimating the actual life, and on the other hand from over-estimating it. Assuming again a business which has a long seasoned period before final retirement, the under-estimating of life of property would impose disproportionally greater amounts of depreciation charges on all of the rate payers excepting the last ones, who are not charged with any depreciation. Further-



more, the last customers enjoy the use of the property and are not required to pay a return thereon, because the rate base has been written off.

In the case of overestimation of life, the equalization of the loss as

between the rate payers and the company cannot be accomplished if, instead of having an infinite life, it retires from business before it has recovered all of its investment.

The sinking fund method also rests on the estimate of life and is subject to limitations similar to the ones applicable to straight line.

Now, let us examine the premises on which our theory rests, in the light of the facts developed from experience. Any actual business would not enjoy the ideal conditions assumed, for the book balances would not remain constant from year to year, nor would the annual additions and retirements be equal. As a matter of fact, experience shows that the retirements in some years are several times the average retirement. Furthermore, in practice, rates cannot be changed from year to year and therefore the Company may be forced to take a loss in those years when retirements are larger than average, and again, in practice a public utility company would not be permitted to make up such a loss by assessing future rate payers. These are most serious limitations to the retirement expense method and constitute a difficult managerial problem for several of the largest utilities in the country. An even greater difficulty than the one already mentioned, would arise upon the retirement of a business; since it would be unable to recover its investment in all the property then in service.

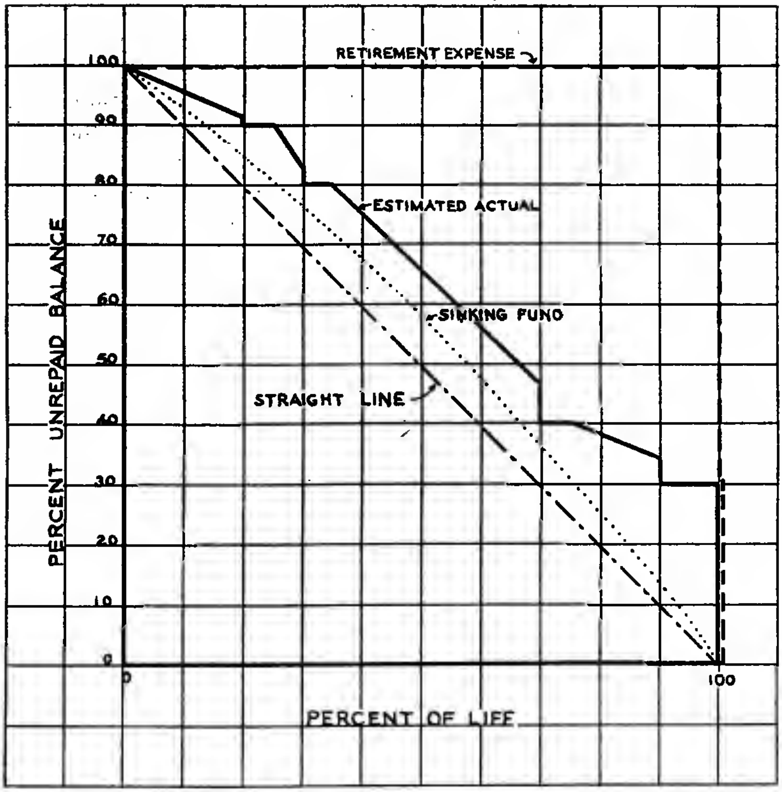
Similarly, the installation expense accounting method would involve the further difficulty of financing the extraordinarily large initial costs of installation as well as those of any future expansion. This method would really reimburse the investors as rapidly as funds were put into the business. Furthermore, the investors would be reduced to the status of controllers and managers, whereas the rate payers would become the owners in fact. This would result from the situation in which the rate payers reimbursed the investors for all money put into the business and as a consequence the rate base would be zero and the allowable return on capital would be zero. The rate payers would then be burdened with the payment of operating expenses alone.

The straight line method and sinking fund method of accounting for depreciation have received considerable attention in a publication bearing the significant title, *Authoritative Definitions of Depreciation and Related Terms,* prepared under the auspices of the General Accounting Committee of the Edison Electric Institute.[11]

Seven references are given defining the straight line method, and five defining the sinking fund method. These are listed at the end of this chapter. Throughout these definitions are found such phrases as "—di-

vided by the number of years of its (property unit) expectation of life"; "—computation of depreciation in which elapsed time is the only factor considered, cost is distributed over the expected life equally as to

COMPARISON OF UNREPAID BALANCE BASED ON VARIOUS METHODS OF DEPRECIATION ACCOUNTING



years—"; "in this method the main factors are the age of the plant and its probable life."

The advocates of straight line or of sinking fund methods usually recognize the difficulties arising from the failure to estimate life correctly where accrued depreciation is used in arriving at a rate base. The earlier applications of either method were based on published lives for

various types of property. It was not expected that the property under consideration would actually experience the predicted life stated in print, but the latter was the best guess that could be made under the circumstances. Such published data were based in part, at least, upon actual experience.

The difficulties to which the use of such life data give rise are obvious and need not be reviewed here. The experienced statisticians, engineers and accountants of regulatory bodies, seeking to force the adoption of the straight line method, have turned to the practice of life insurance actuaries for guidance. In general, life insurance experience tables rest on the assumption that in a given homogeneous population, the life experience of a random sample of sufficient size will fairly represent the experience to be expected of any other sample of at least the same size drawn from the same population. The past is assumed to be the best guide to the future. It should be recognized that the problem of life insurance of human beings has many aspects which are very different from estimates of life expectancy with respect to business assets. It is true that the medical art has made certain contributions toward longevity in certain areas, that ravages of diseases have in some instances been reduced, that the advent of the automobile and the increased death-dealing efficiency of wars have tended to offset the tendencies toward increased life spans. Nevertheless, excepting for certain cataclysmic occurrences, life expectancy has remained about the same. It is certainly not far from the truth to assert that there have been no improvements in the art of building human beings; better, different and improved models have not appeared in sufficient number to cause the premature "retirement" or death of existing types.* In general, changes in the market, in the demand for services, in the adequacy, etc., have not caused the expiration of life. In other words, life as reflected in the actuarial studies, has, in general, been as long as the effects of wear and tear, disease and accidents have dictated. When the methods, applicable to human beings, are transferred to physical property, it is questionable whether the same results may be expected with respect to the accuracy of average life expectancy. Entirely apart from the difficulty of collecting the necessary historical data, even if these are still extant, the most serious aspect of the problem is the assumption and subsequent verification that the type of property, for which historical data are available, is

* The aspirations of certain groups which when bolstered by sufficient force cause the destruction and death of other groups are not indicative of the relative worths of the two. The times of occurrence of such cataclysms are not predictable.

the same in all essential characteristics, as the present property under examination. Frequently the name and function are the only points of similarity. For example, boilers for the generation of steam, constructed in the early 1900's, have little in common with modern boilers, excepting for the type name, boiler, and the function, generation of steam. Life span data with respect to early boilers throw little light on what might be expected of any modern type. Likewise cedar poles subjected to certain types of butt treatment in the years up to 1930 would yield life span data somewhat different from poles subjected to the new and improved treatments. The essential assumptions for the successful application of actuarial methods in the determination of life expectancy of human populations are not generally valid in the field of physical property.[12]

In describing the application of the straight line method in a case before the New York Public Service Commission[12b] the Commission's engineer stated that:

> The average service life is calculated from a Life Table such as insurance actuaries use. A Life Table is a statement of the proportion of individuals born or units put in service *the same year* which survives to each age until all members of the group are gone. A Life Table can be formed in several ways:
>
> If the history of a completed generation is available, the number surviving to each age is known and the table can be written with ease. It is obvious that the calculations of the average life expectancy at any age is then a simple matter.
>
> (Further quotations from this source are given in appendix.)

This is the central idea on which the straight line theorists rest their case. But the data are, for the most part, insufficient. Types of property bearing particular names, such as watthour meter, distribution transformer, oil circuit breaker, etc., actually have undergone far-reaching changes in design, so that data concerning ages at times of retirement of transformers built in 1904 are hardly comparable to similar data in respect of transformers built in 1925. Furthermore, the reasons for the retirements of transformers built prior to 1915 may not be valid for transformers built in 1927. Factors which have caused certain retirements in the past may not recur. Thus the retirement of a large number of poles or of a large quantity of pipe, due to a subway construction or street widening project, should not be expected to occur again at the same point in the life history of present poles. The most serious problem at present, but which may be overcome in the future, is the

utter lack of sufficient data regarding dates of installation, dates of retirement and reasons therefor.

The problem of accounting for depreciation, or loss in value due to all causes, is one of injecting a value element into a system of bookkeeping based on costs. The question is not whether one or the other method is simpler, more direct or easier to demonstrate, but is whether the particular method measures loss in value from all causes, as nearly as possible.

Hypothesis No. 1—Depreciation can be Measured by Straight Line Accruals

Time is the controlling factor. Actuarial studies of data from the past form the basis for estimates of average lives of various classes of physical property. Average life is taken to be the average number of years from date of installation, or acquisition, to the date of retirement, for any cause, of all property within one class. The observed fact is that total depreciation occurs by the time of actual retirement. Depreciation then is assumed to occur in equal increments of time. The physical property when first acquired, or installed, is represented by a finite number of dollars of investment, and the expiration of the physical property necessitates that these dollars be written off the books in accordance with legal and accounting conventions. At any time before retirement, it is assumed that the extent of depreciation may be reasonably measured by the product of the original number of dollars times the ratio of age to life. The actuality is depreciation. The dollars expressing it constitute the implements of thought expressed. In other words, depreciation may be a physical fact or an economic concept or both. Dollars are intended as measures of the extent of depreciation.

The basic assumptions or conventions on which this method rests are:

1. Actual average life can be predicted within reasonable limits.
2. Past experience with property falling in a particular category is the least unsatisfactory guide for the prediction of life of a particular type of property.
3. Since life expires in apparent equal increments of time from birth to death, and since there is no depreciation at birth and total depreciation at death, the depreciation proceeds with equal increments of time.
4. Because dollars of investment are totally present at time of acquisition and are totally written off when property is retired, dollars are good units by which depreciation may be measured with the passage of time.

The foregoing states the hypothesis and the assumptions therein.

For the purpose of testing the hypothesis two identical properties are assumed, for which the accounting practices entail straight line depreciation. If one company pursued a vigorous maintenance policy while the other permitted its property to deteriorate, the property of the first company will remain in good condition whereas that of the second will be in relatively poor condition some years after the commencement of operations. Disregarding other factors, the reserves for depreciation in both cases will indicate the same accrued depreciation, but the facts are at variance with this result. It is true, of course, that the more sophisticated advocates of straight line depreciation would have readjusted the annual depreciation charges by changing the originally assumed lives. But this is hardly adhering to a straight line policy. In fact, it is a recognition of the necessity of relying on observed conditions and actualities of the present, for the purpose of modifying a mathematically determined set of average lives.

Hypothesis No. 2—The Use of Retirement Expense Method in Lieu of Depreciation

As property is retired it is written off the books. The annual provision for expected retirements represents all the depreciation which exists. Thus it is assumed that the property in service at any time has suffered no depreciation and that as fast as depreciation occurs property is retired. One of the difficulties in developing the retirement expense method rests on the fact that there is no compulsion to retire property. As a consequence a management which has taken into account the various economic effects might retire property from time to time as the facts in respect of physical deterioration and decline of economic usefulness became known. On the other hand a management might keep in service deteriorated and obsolete equipment and yet find that the rate base was not different from that of the more conservatively managed company previously described.

Hypothesis No. 3—Depreciation Can be Measured by Sinking Fund Reserves

Except for the compound interest factor, the sinking fund method is based on the same conditions as govern the straight line method. The sinking fund method is characterized by the facts that the rate base remains undiminished, the annual credits to the reserve are constant, and

the growth of the reserve, while constant in terms of credited dollars is varying because of the interest accruals. Both straight line and sinking fund involve the prediction of lives and therefore both require readjustments or new predictions from time to time to keep step with actualities.

Conclusion

It is necessary to express the economic and certain of the physical changes in property in terms of money. The goal is to express these changes in dollars as the changes occur or become known.

It is recognized that great difficulty is encountered in attempting to express the facts of deterioration, obsoleteness, inadequacy and changes in demand in terms of dollars. Perhaps it would not be too far afield to liken the attempt to the one in which the early natural philosophers attempted to describe and measure temperature.

Man, in his attempt to communicate his experiences, devised linguistic symbols "hot" and "cold" or their equivalents. These symbols, being limited in implication, were extended by the use of the comparative and superlative forms. The crudeness of the language tool is obvious.

According to Professor W. Schüle:[13]

> It is possible to decide with some certainty, by sense of touch, in the case of two substances, whether and by how much the one is warmer or colder than the other. In bodies of different material, however, this method in general fails when the temperature difference is not high. Naturally the sense of touch conveys no quantitative measure of these differences. (Page 41)

We might profitably trace the steps to the present comprehension of that elementary experience. For our purposes here, however, it will suffice to state how the modern physicists define temperature. It is "the measure of the mean kinetic energy of the motion of translation of the molecules of matter."

Professor Schüle states that:

> A measure of temperature, which is independent of the nature of heat, is afforded by the changes in the state of bodies when receiving or giving up heat. (Page 41)

The thermometer is a device which reacts to the statistical average impingements of the molecules in such a way as to cause a reading on an arbitrarily divided scale or dial to be repeated again and again whenever the conditions within the medium under consideration are the

same in respect of the kinetic energy of the motion of translation of its molecules. From the same authority we learn that:

The basic scale of temperature is obtained by means of the expansion of gases, since, of all substances, these show the greatest change in volume when heated. (Page 42)

The establishment of the Fahrenheit scale for measuring temperature is an interesting illustration of how measurement units become universally understood, making possible the recording of experience despite the arbitrary magnitude of the selected unit. Fahrenheit being a cartographer and geographer was equipped with precision instruments for dividing circles into 360 equal parts. The laboratory techniques which produced the coldest and most easily reproducible combination of substances was found to be melting ice and rock salt and the hottest was found to be boiling water. Fixing these two points as his scale limits, Fahrenheit is alleged to have found 360 divisions to be too small for marking purposes; so he chose the half circle or 180 divisions as his standard.

The experience with depreciation, like that early experience with temperature, has caused us to select certain types of facts which permit us linguistically to identify that experience and to discuss it in qualitative terms. We are now looking for a definitive statement of what depreciation is and a scientific means by which depreciation may be measured. The methods of straight line, sinking fund, and retirement expense are attempts to measure depreciation. It is here contended that at this stage of our knowledge some of the facts of depreciation may be measured objectively, while others must still depend on subjective standards.

Our efforts should constantly be directed toward the establishment of a method and scale for measuring our depreciation experience.

APPENDIX

The following material has been taken from "Summary of Definitions Covering Depreciation and Related Terms," prepared under the auspices of the General Accounting Committee of the Edison Electric Institute, July 29, 1939.

STRAIGHT LINE METHOD

Proceedings of the American Society of Civil Engineers. Volume XLII, No. 10. December, 1916. Report of the Special Committee to

Formulate Principles and Methods for the Valuation of Railroad Property and Other Public Utilities. Page 1926.

Straight Line Method. This method makes provision from the earnings for equal annual depreciation allowances of such size, for any property unit, that the sum of the allowances, without interest, will equal the cost, less salvage, of each unit when it reaches the end of its estimated service life. The annual depreciation allowances by this method are each equal to the cost, less salvage, of the property unit divided by the number of years of its expectation of life.

ACCOUNTING TERMINOLOGY. Preliminary Report of a Special Committee on Terminology of the American Institute of Accountants. Publishers: American Institute Publishing Co., Inc., New York. Second Printing March, 1934. Page 51.

Straight Line Method. The computation of depreciation in which elapsed time is the only factor considered. Cost is distributed over the expected life equally as to years, the entry being a charge to operations and a credit to depreciation reserve. The amounts so written off are not actually segregated and invested for the depreciation fund but are left in the general funds of the business. This is the plan prescribed by income-tax regulations.

CLARK'S FERRY CO. v. COMMISSION (1934), 291 U.S. 227, 240, quoting from the opinion below, 106 Pa. Superior Ct. 49; 165 Atl. 281.

Straight Line Method. "The straight line method is often used for short-lived structures, or plants of a character that they can be restored from time to time to the original condition of efficiency. *** It is not so fair or equitable when applied to a long-lived structure or one that is disintegrating gradually and continuously and not capable of being restored to its original condition."

SOUTHERN BELL TELEPHONE & TELEGRAPH CO. v. RAILROAD COMMISSION OF SOUTH CAROLINA, 5 Fed. (2d) 77, P.U.R. 1926 A. Pages 6, 44.

As the court understands it (the straight line method), in this method the main factors are the age of the plant and its probable life. The age is ascertained from the history and inspection, and the probable life from studies and inspection by experienced men. It is assumed that the depreciation is uniform and constant, and defendants' expert stated that it was used in those plants where the depreciation is uniform and constant. Having ascertained the age of the property and its probable life, a definite rule is obtained whereby to fix exactly the amount of depreciation at any particular time. Thus, as explained by the witness, if the property is five years old and its probable life is ten years, the depreciation is 50 per cent. It is obvious that this rule is highly theoretical and may in many cases lead to grossly erroneous results. *** In physical examination the depreciation rests upon esti-

mates and opinions of witnesses. But the theoretical methods mentioned rest not only upon estimate and opinion, but upon the correctness of a theory which is of doubtful correctness in many cases, and which we know cannot be correct in all cases.

Public Service Commission. State of New York. Uniform System of Accounts for Electric Corporations. Effective January 1, 1934. Definition, page 9.

39. "Straight line method" as applied to depreciation accounting means the plan under which the service value (see definition 38) of property is charged to operating expense or other accounts and credited to the depreciation reserves through equal monthly charges as nearly as may be during its service life.

National Association of Railroad and Utilities Commissioners. Report of Special Committee on Depreciation. Submitted to the National Association of Railroad and Utilities Commissioners at its 50th Annual Convention in New Orleans, Louisiana, November 15–18, 1938. Page 15.

Straight Line Method. The straight line method is one in which the service of plant is spread over its life, as nearly as practicable, in equal annual amounts. To illustrate, if property costs $1,000 and is expected to have a life of 10 years, the annual charge to expenses, assuming no salvage or cost of removal, would be $100. This amount would be credited annually to depreciation reserve and at the end of 10 years would accumulate to $1,000, an amount equal to the cost of the property to be retired.

Kohlhepp, C. E. Wisconsin Public Service Corporation, Milwaukee, Wisconsin. The 1938 Depreciation Problem, Edison Electric Institute Bulletin (December, 1938). Pages 517–8.

The straight line method has been defined as one in which the service value of plant is spread over its life, as nearly as practicable, in equal annual amounts.

SINKING FUND METHOD

Proceedings of the American Society of Civil Engineers. Volume XLII, No. 10. December, 1916. Report of the Special Committee to Formulate Principles and Methods for the Valuation of Railroad Property and Other Public Utilities. Page 1926.

Sinking Fund Method. This method makes provision from the earnings for equal annual depreciation allowances of such size for any property unit that if these allowances are invested in a sinking fund they will, with accumulations of interest compounded annually, amount to the cost, less salvage, of the unit at the time when it reaches the end of its estimated service life. In applying this method, it has been common practice to compute annuities upon a basis of

original or reproduction cost and total service life but the method applies as well if, at any time, actual value and remaining service life are used.

NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS. Report of Special Committee on Depreciation Submitted to the National Association of Railroad and Utilities Commissioners at its 50th Annual Convention in New Orleans, Louisiana, November 15–18, 1938. Page 15.

Sinking Fund Method. The sinking fund method is a procedure whereby an annuity is established, which, with compound interest at a certain stipulated rate, will equal the service value of the plant at its retirement. To illustrate, if property costs $1,000 and is expected to have a life of 10 years, there would be required, under a 6 per cent sinking fund method, an annuity of $75.87, which, with interest at 6 per cent compounded annually on the accumulated annuities, would exactly equal the cost of the property at its retirement.

VALUATION OF PUBLIC SERVICE CORPORATIONS, by Robert H. Whitten, Ph.D., and Delos F. Wilcox, Ph.D. (Second edition, 1928) Page 1826, quoting from Report of Depreciation Section of Interstate Commerce Commission, March 10, 1923.

"The sinking fund method, like the annuity method, involves complicated interest computations. It also recognizes a uniform annual charge to operating expenses, but its application requires that the annual allowances shall be actually set aside in a special fund or invested in approved securities which, with interest compounded, shall during the life of the depreciating property accumulate a sum equal to the service value."

As in the annuity method, the rate of interest is one of the controlling factors in determining the annual charge. It is necessary to find a sum which will, when added to the interest accumulations, equal the service value at the expiration of the life period. It should be remembered, however, that under the annuity method the interest is hypothetical, while under the sinking fund method the interest may be actually earned. A sinking fund increases not merely by direct contributions but also by the interest accumulations of these contributions, and it therefore follows that the longer the sinking fund is in operation the larger the interest accumulations will be. At first thought it may appear that the direct cost of maintaining such a fund grows smaller with the increase in the number of years, but the fallacy of this is apparent when it is remembered that the contributions to the fund are wholly withdrawn from the business where they might be employed at a greater profit than could reasonably be expected from the sinking fund. However, under the sinking fund method the interest accrued is applied to reduce the loss of property

consumed in operation which otherwise is chargeable to operating expenses.

BRUNDAGE, PERCIVAL F., DEPRECIATION—AN OLD SUBJECT WITH A NEW IMPORTANCE. Article in Harvard Business Review. Spring, 1935. Page 338.

Sinking Fund Method. Another basis for the depreciation computation is the sinking fund method. This contemplates the provision of a lesser annual amount than the straight line method, on the assumption that cash in the respective amounts will be deposited in a separate fund which will earn interest and will at the end of its useful life amount to the original cost or estimated replacement value of property unit. The interest rate to be used is subject to uncertainty, however, and in practice it is very seldom that the cash is actually set aside, since it is usually needed for the business. If expended for property additions, it will be necessary to provide a charge against operations for depreciation on the additions and also for a return on the accumulated amounts supposed to be set aside in the fund; otherwise the reserve will be insufficient at the expiration of the useful life of the original unit.

Once the total provision for the period has been determined, the amount is charged to operations and credited to a reserve for depreciation. As soon as replacements are required, the question arises as to their proper accounting treatment. The whole cost of the new unit might be capitalized, and the cost of the unit replaced credited to the asset account and a charge made to the depreciation reserve, either (1) in the entire amount, or (2) only such portion as has already been covered by depreciation provisions and the balance charged against profit and loss; or the cost of the replacement might be charged against the depreciation reserve without disturbing the asset account. There are sound arguments to support any of these treatments so long as the policy adopted meets the particular needs of the business, is consistently followed, and the depreciation rates were fixed in relation to it.

PUBLIC UTILITY VALUATION, by Bauer and Gold. (Bauer) Formerly Lecturer on Public Utility Regulation at Cornell, Princeton & Columbia; also Director American Public Utilities Bureau. (Gold) Member N.Y. Bar; Lecturer Public Utilities at C.C.N.Y. Page 205.

The sinking fund method starts with equal annual sums based on estimated service life of a unit, but these are so computed that the aggregate base annual amounts plus compound interest at a given rate will equal the cost of the unit. This method, as usually conceived, presupposes separate investment of the amounts reserved which with interest accrual will create a fund equal to the original cost at expiration of the estimated life.

CHAPTER III

IMPLICATIONS OF THE WORD *DEPRECIATION*

A Treatment of the Concept as an Element of Total Cost of Production

It has been suggested that depreciation might be treated as a problem of costs.* The operation of a steam power plant will serve to illustrate the point. It will be assumed that a large quantity of coal, bought at a fixed price, is held in storage. As the plant generates electrical energy, coal is moved from storage to the feeder hoppers. The rate of coal consumption is not directly proportional to the station energy output, because the overall efficiency does not remain constant as the output varies. If the plant always operates at its maximum efficiency, its consumption of coal, per unit of production, will be a minimum and will remain unchanged. The consumption per unit of production depends on the point on the efficiency curve at which the plant is operating. Coal is not consumed in equal increments with the passage of equal units of time unless the very special condition of unvarying load prevails.

Coal lying in storage reacts with the oxygen of the air permeating the pile and a slow combustion rate is experienced. In other words, the coal in storage is deteriorating. The total cost of the coal stored represents an investment in potential heat, and upon arrival in storage, the available potential heat could be determined within practical engineering limits. With the passage of time the remaining coal will have lost a part of the original potential heat, due to a number of factors which govern the rate at which this relatively slow combustion takes place. But for the purpose of this example, it is sufficient to recognize, that if the rate be constant, then the accumulated lost potential heat might be determined by multiplying the elapsed time by this rate. In other words, the heat consumed in storage might be related to the passage of time by assuming a constant rate. The total consumption of heat, however, is the sum of the heat lost in storage and the heat developed in the furnace. It is

* Depreciation might be viewed as a proper item for consideration in casting up the cost of producing a unit of production and analyzed as an element of such cost. Thus it becomes a problem of apportioning to each unit produced its appropriate share of asset value diminution. "Appropriate share" should not necessarily be taken to mean *proportional* to the units produced.

evident, therefore, that the total consumption is not a simple function of time.

Coal is bought in terms of weight, although heat is the real object of the transaction. The total consumption may be represented in terms of a portion of the original total cost of all the coal in storage. The worth of the remaining heat or coal, measured in units of original cost, is the difference between the total original cost and the sum of the heat consumed in the furnace and the lost heat of the remaining coal, both expressed in the same cost units.

The history of the operations of other plants, or the passage of time, throws little light on the expected "life" of the coal pile; its "age" related to this "life" has little bearing on the heat lost or the heat remaining, whether these be expressed in terms of British Thermal Units or in terms of dollars.

The loss in value of the original investment in heat, in terms of original cost, has followed no recognized function of time, but has resulted from the interaction of numerous influences.

The estimate of the amount of remaining potential heat from measurements respecting the quantity of remaining coal and the heating value of a unit thereof, is to be preferred to estimates of life, age and the rate of consumption.

The cost of a unit of production, say a kilowatt-hour, is the sum of numerous costs including the cost of the total heat consumed both in the furnace and in the storage pile. There are a number of techniques applicable to the determination of the portion of total cost attributable or allocable to the unit of production. These techniques involve most of the facts and may be carried out with engineering skill and accuracy.

The depreciation of physical property in terms of original costs is analogous to this consumption of heat expressed in terms of cost. The great distinction rests in the absence of a refined technique for determining the magnitude of the "consumption" of physical property.

The development of a technique which will permit the determination of the overall effects of all of the causes of depreciation, expressed in terms of cost, is the objective. To make assumptions as to life and age, for the purpose of short-cutting the effort, where this requires the disregarding of realities, is inconsistent with the trend toward realistic accounting and rate regulation.

CHAPTER IV

DEPRECIATION AS A FACTOR DETERMINING THE RELATIONS OF A PUBLIC UTILITY WITH ITS CONSUMERS AS A WHOLE AND AS GROUPS IN DIFFERENT PERIODS OF TIME

According to Professor John R. Commons:[14]

> The common law is historic custom, precedent, and the ancient law of the land; equity is conscience, reason, and the law of God or nature. (Page 233.)

What an equitable relationship is depends upon moral and ethical standards, which are themselves unique in respect of any particular social organization. In order to avoid a protracted discussion of moral and ethical standards, the discussion will proceed within the framework of the social and legal system in operation in the United States at this time.

The problem of depreciation may be most readily studied by dividing it into two sub-problems, the one dealing with investors as a group versus the rate payers as a group, the other dealing with earlier and later groups of rate payers.

The investors have the choice of consuming their funds to satisfy their own wants, or of investing these funds in some business enterprise, which may result in a profit or in a loss depending on numerous and unpredictable influences. It is common experience to find that there is no fixed relationship between an investment and any return thereon. In the public utility business, due to its character as a natural monopoly, society has adopted regulation as a substitute for competition. Theorists of the "prudent investment" school will not agree with this view.* The investors in such a business have taken the choice of placing their funds in a regulated enterprise rather than placing these in a competitive business where the limits to the possible profits are not set by rate regulation. Once the choice is made the owners will seek to charge rates sufficient to pay operating expenses as well as a return *on* the investment plus the return *of* the investment. Of course, this view of equity to public utility investors is not the presently accepted legal philosophy, for the

* What theorists of the prudent investment school will deny is that the substitute should be highly imitative.

courts under the "fair value" doctrine expressly decline to accept a return *on* and the return *of* actual investment as criteria of fairness.* This apparent conflict need not be troublesome because the assumption has been made here that actual investment less accrued depreciation and "fair value new" less accrued depreciation, yield the same rate base.

The choice of investing in a public utility enterprise is undoubtedly based on the desire to enjoy relative security and constancy of return. The requirements of equity or fairness are (1) that the capital supplied by the investors and dedicated to the public service must be returned by the rate payers and (2) that while the capital is in the public service the owners are entitled to a return thereon. It follows that whatever the original investment may have been, the rate payers need only pay a return on such investment until they have reimbursed the owners. Thus, as the reimbursement proceeds, the difference between the original investment and the total reimbursements is the basis on which the rate payers must pay a return.

A return *on* the investment is really rent paid for its use; a return *of* the investment is analogous to the repayment of a loan. A consideration of the implications of equitable rate payer–owner relationships leads to the conclusion that the reason why a return must be paid on the investment is that it represents capital invested and thus withheld from its lawful owners who might have chosen to lend or employ it elsewhere. Consequently, the equities are fully balanced when the rate payers have discharged their obligations by returning the investment and by paying for its use during the time it was devoted to their service.

The rate payers pay for the use of the property but they too have a choice, that of immediately investing their own funds in a plant or of renting one and obligating themselves to secure the investor against loss of his investment. This is true whether the small rate payers act as a group or the large ones act independently. It is not true in respect of individual small rate payers.

If these relationships could be renegotiated at short intervals of time, the rate payers could consider, from time to time, substitute facilities for their needs. Under such circumstances, the rate payers would probably be unwilling to pay as much for the use of a worn or deteriorated plant as for a new and more efficient one. Furthermore, if substitutes were available on the market to satisfy the same wants in a less expensive

* See the leading cases on Present Fair Value and on Rate of Return listed in the bibliography.[15]

way, the rate payers would undoubtedly turn to them. Because these negotiations cannot be reopened at short intervals of time, but must be concluded on a long period basis, it seems essential to consider the effects of deterioration or wear and tear, inadequacy, obsoleteness in respect of the basis for renegotiation. The choice as to the manner of repaying the capital to the owners should rest with the rate payers as a group for, if they wish to use a worn and partially outmoded plant, they may do so by paying a return to the owners on the whole original investment and may return that total investment at the time when the property expires. On the other hand, the rate payer may prefer to reimburse the owners as depreciation occurs. It is contended here that consistent treatment of return and depreciation leads to equitable results, as between the investors and the rate payers as a group, irrespective of the manner in which the repayments of capital are made. At this point, no consideration is given to the requirements of fairness to different year groups of rate payers.

Table 1 and the explanation accompanying it illustrate what is meant by equitable and consistent treatment of the owner–rate payer relationships. Any system which will repay to the owners the original investment at the end of a period while requiring the rate payers to pay a return on or hire of the unreturned balance or capital remaining in the enterprise is equitable.

The matter should be allowed to rest here, except for the fact that the same rate payers do not continue as customers of the business; instead old ones drop off and new ones are acquired. From this follows the further equitable consideration that the rate payers of each year should repay their proportionate share of the capital consumed in their service. The adoption of an arbitrary mathematical repayment schedule makes the rate payers in some years repay too much and others repay too little. The method of observed or estimated actual depreciation attempts to equalize the burden so as to keep step with events as they actually occur.

Since investors frequently buy and sell their securities, the depreciation policies should be well defined and made clear to prospective investors; certainly they cannot rely on stock market price adjustments to guide them. Perhaps their informed actions as buyers and sellers should rather determine the price adjustments that occur in the market. Any consistent depreciation method may be fair to the investors as a whole, but some methods (like the retirement expense method) while they *would* be fair to the investors if the company could recapture its capital

outlays, even though it went out of business, may break down for financial or other practical reasons.

There are appended several tables which demonstrate the consistent treatment of annual depreciation and accrued depreciation which bring about the results just discussed.

APPENDIX

Explanation of Table 1 which is intended to demonstrate what is meant by consistent owner–rate payer relations.

Column (a) of Table 1 is a succession of years throughout the life of a property. Column (1) shows the capital repayments, or depreciation payments, made by the rate payers to the owners. The repayments at the end of each year are at random, that is these follow no mathematical law except that their sum equals the original investment. Column (2) shows the unrepaid balance at the beginning of each year and is zero at the beginning of the $(L + 1)$ year. Column (3) shows the dollars of return, determined by multiplying the rate r by the unrepaid balance or capital remaining in the utility enterprise shown in Column (2). Column (4) is shown for completeness only, and indicates the dollars of return each year on the capital returned by the rate payers and reinvested by the owners in another business, or in the same business. The annual hire or rent paid to the owners by rate payers on the capital still remaining in the enterprise and on capital repaid and reinvested is obviously Ar. This is shown in Column (5). Column (6) is the return shown in Column (5) compounded for L years. The owners at the end of L years will have received both A, the capital originally invested, and $A[(1 + r)^L - 1]$. The compounded return or the total sum so received is, therefore, $A + A[(1 + r)^L - 1]$ or $A(1 + r)^L$.

The present worth of a sum S received after L years and at a rate of return or rent of r per annum, is $\dfrac{S}{(1 + r)^L}$. If S is equal to $A(1 + r)^L$ then the present worth of $\dfrac{A(1 + r)^L}{(1 + r)^L} = A$. Which means that A, the original capital sum, if invested at compound annual interest, r, would equal S or $A(1 + r)^L$ at the end of L years.

This is a general statement which would be equally true if Column (1) were so designed that A_1, A_2, A_3, etc., followed any predetermined law or function. This means that, if the table is properly prepared as shown, A_1, A_2, A_3, may be unrelated, or may follow sinking fund, straight line or any other mathematical formulation without disturbing the equitable result indicated. This is the test for equitable depreciation practice previously referred to.

Table 2 illustrates the principle of consistency as applied to a property where straight line depreciation accounting is utilized.

Column (2) indicates the depreciated investment which constitutes the rate base at the beginning of each year.

Column (3) represents the equal annual depreciation payments in accordance with the straight line method.

Column (4) represents the accrued depreciation at the end of each year.

Column (5) represents a 7% return on the depreciated investment or rate base.

Column (6) represents the total payments by the rate payers of return on the rate base plus annual depreciation charges.

Column (7) represents the present worth factors based on 7% return.

Column (8) is the product obtained by multiplying the return on the rate base plus the annual depreciation payments by the respective present worth factors.

It should be noted especially that the total of Column (8) is approximately equal to the total of Column (3) and to the original investment which is the test of equity referred to in this chapter.

Turning next to **Table 3**, which is an examination of the same problem except on the basis of sinking fund depreciation accounting, without discussing each of the column headings which are analogous to Table 2, it should be noted that the investment remains constant throughout the life of the property and that the apparent rate base likewise remains undepreciated.

The total of Column (8), which represents the sum of the present worths of the annual depreciation charges plus the returns on the undepreciated rate base, equals the original investment.

Table 4 represents the situation with respect to a property to which the estimated actual depreciation accounting method has been applied.

Notice especially that Column (3) indicates the lack of uniformity in making annual depreciation charges.

Table 5 represents a property to which retirement expense accounting has been applied.

All of these tables indicate that consistency of treatment results in equitable rate payer–owner relationships.

The algebraic proofs of these tables follow them in sequence.

TABLE 1

Let A = A capital sum

r = Rate of return on remaining value as well as on the reinvested repayments

L = Number of years until final payment is made

n = Any year between 0 and L years

A_n = Any variable or constant amount repaid in a year, n

ΣA_n = Sum of repayments made up to and including the nth year

(a) Year	(1) Repayment at End of Year	(2) Unrepaid Balance Beginning of Year	(3) Return on (2)	(4) Return on Reinvestment of (1)	(5) Sum of Returns (5) = (3) + (4)	(6) Compounded Amount of (5) After L Years
1	A_1	A	$(A)r$	0	Ar	$Ar(1+r)^{L-1}$
2	A_2	$A - A_1$	$(A - A_1)r$	$(A_1)r$	Ar	$Ar(1+r)^{L-2}$
3	A_3	$A - (A_1 + A_2)$	$[A - (A_1 + A_2)]r$	$(A_1 + A_2)r$	Ar	$Ar(1+r)^{L-3}$
n	A_n	$A - (\sum A_n - A_n)$	$[A - (\sum A_n - A_n)]r$	$(\sum A_n - A_n)r$	Ar	$Ar(1+r)^{L-n}$
L	A_L	$A - (\sum A_L - A_L)$	$[A - (\sum A_L - A_L)]r$	$(\sum A_L - A_L)r$	Ar	$Ar(1+r)^{L-L} = Ar$
Sum	A					$A[(1+r)^L - 1]$*

At the end of L years the amount available, adding liberated "Reinvested Returns," sum of Col. (1) and the sum of Col. (6), equals

I.
$$A + A[(1+r)^L - 1] = A(1+r)^L = S$$

II. Present worth of S received after L years at a rate of interest r

$$pw \text{ of } S = \frac{S}{(1+r)^L} = \frac{A(1+r)^L}{(1+r)^L} = A$$

* Sum of Col. (6)
$$= Ar + \cdots Ar(1+r)^{L-n} + \cdots Ar(1+r)^{L-2} + Ar(1+r)^{L-1}$$
$$= \frac{Ar[(1+r)^L - 1]}{(1+r) - 1}$$
$$= A[(1+r)^L - 1]$$

TABLE 2

Straight Line Depreciation: Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—Straight Line Annual Depreciation and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Depreciated Investment Beginning of Year	(3) Annual Depreciation	(4) Accrued Depreciation End of Year	(5) 7% Return on Column (2)	(6) Return and Depreciation Columns (3) + (5)	(7) Present Worth Factor @ 7%	(8) Present Worth Columns (6) × (7)
1	$1,000.00	$ 100.00	$ 100.00	$ 70.00	$ 170.00	.9345794	$158.87849
2	900.00	100.00	200.00	63.00	163.00	.8734387	142.37050
3	800.00	100.00	300.00	56.00	156.00	.8162979	127.34247
4	700.00	100.00	400.00	49.00	149.00	.7628952	113.67138
5	600.00	100.00	500.00	42.00	142.00	.7129862	101.24404
6	500.00	100.00	600.00	35.00	135.00	.6663422	89.95619
7	400.00	100.00	700.00	28.00	128.00	.6227497	79.71196
8	300.00	100.00	800.00	21.00	121.00	.5820091	70.42310
9	200.00	100.00	900.00	14.00	114.00	.5439337	62.00844
10	100.00	100.00	1,000.00	7.00	107.00	.5083493	54.39337
		$1,000.00		$385.00	$1,385.00		$999.99994

TABLE 3

Sinking Fund Depreciation: Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—7 Per Cent Sinking Fund Annual Depreciation and 7 Per Cent Return on Investment—Equal Annual Payment Method

(1) Year	(2) Investment	(3) 7% Sinking Fund Factor	(4) Annual Depreciation	(5) 7% Return on Investment	(6) Return and Depreciation Columns (4) + (5)	(7) Present Worth Factor @ 7%	(8) Present Worth Columns (6) × (7)
1	$1,000.00	.072377	$ 72.377	$ 70.00	$ 142.377	.9345794	$133.06261
2	1,000.00	.072377	72.377	70.00	142.377	.8734387	124.35758
3	1,000.00	.072377	72.377	70.00	142.377	.8162979	116.22204
4	1,000.00	.072377	72.377	70.00	142.377	.7628952	108.61872
5	1,000.00	.072377	72.377	70.00	142.377	.7129862	101.51283
6	1,000.00	.072377	72.377	70.00	142.377	.6663422	94.87180
7	1,000.00	.072377	72.377	70.00	142.377	.6227497	88.66523
8	1,000.00	.072377	72.377	70.00	142.377	.5820091	82.86470
9	1,000.00	.072377	72.377	70.00	142.377	.5439337	77.44364
10	1,000.00	.072377	72.377	70.00	142.377	.5083493	72.37724
			$723.770	$700.00	$1,423.770		$999.99639

Note: Total of Column (4) $723.77 is the sum collected from the rate payers as repayments of capital. The sum of repayments invested each year at 7% is equal at the end of 10 years to $1,000.00. In other words, the rate payer instead of withholding return of capital till the end of the 10th year and thereby earning a return on that amount for 10 years, repays to the owners in equal installments an amount which, if invested at the same rate of interest, will equal the capital sum of $1,000.00 at the end of 10 years. It follows that the situation is identical with the repayment of a lump sum at the end of 10 years and thus justifies the payment of qua

TABLE 3A

A Restatement of Table 3: Sinking Fund Method

(1) Year	(2) Original Investment	(3) Depreciated Investment	(4) Annual Depreciation	(5) 7% Return on Depreciated Investment	(6) Return Plus Annual Depreciation	(7) Present Worth Factor @ 7%	(8) Present Worth Columns (6) X (7)
1	$1,000.00	$1,000.000	$ 72.377	$ 70.000	$ 142.377	.9345794	$133.062611
2	1,000.00	927.623	72.377	64.934	137.311	.8734387	119.932741
3	1,000.00	855.246	72.377	59.867	132.244	.8162979	107.950499
4	1,000.00	782.869	72.377	54.801	127.178	.7628952	97.023486
5	1,000.00	710.492	72.377	49.734	122.111	.7129862	87.063458
6	1,000.00	638.115	72.377	44.668	117.045	.6663422	77.992023
7	1,000.00	565.738	72.377	39.602	111.979	.6227497	69.734889
8	1,000.00	493.361	72.377	34.535	106.912	.5820091	62.223757
9	1,000.00	420.984	72.377	29.469	101.846	.5439337	55.397472
10	1,000.00	348.607	72.377	24.402	96.779	.5083493	49.197537
	Sub-Total		$723.770	$472.012	$1,195.782		$859.578473
10	Balance theoretically withheld until end		276.230		276.230	.5083493	140.421327
	Total		$1,000.000		$1,472.012		$999.999800

TABLE 4

Estimated Actual Depreciation: Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—Variable Annual Depreciation and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Investment	(3) Annual Depreciation	(4) Accrued Depreciation	(5) Depreciated Investment Beginning of Year	(6) 7% Return on Column (5)	(7) Return and Depreciation Columns (3) + (6)	(8) Present Worth Factor @ 7%	(9) Present Worth Columns (7) X (8)
1	$1,000.00	$.......	$.......	$1,000.00	$ 70.00	$ 70.00	.9345794	$ 65.42056
2	1,000.00	100.00	100.00	1,000.00	70.00	170.00	.8734387	148.48458
3	1,000.00	100.00	200.00	900.00	63.00	163.00	.8162979	133.05656
4	1,000.00	....	200.00	800.00	56.00	56.00	.7628952	42.72213
5	1,000.00	....	200.00	800.00	56.00	56 00	.7129862	39.92723
6	1,000.00	....	200.00	800.00	56.00	56.00	.6663422	37.31516
7	1,000.00	400.00	600.00	800.00	56.00	456.00	.6227497	283.97386
8	1,000.00	....	600.00	400.00	28.00	28.00	.5820091	16.29625
9	1,000.00	100.00	700.00	400.00	28.00	128.00	.5439337	69.62351
10	1,000.00	300.00	1,000.00	300.00	21.00	321.00	.5083493	163.18012
		$1,000.00			$504.00	$1,504.00		$999.99996

TABLE 5

Retirement Expense: Present Worth of Annual Depreciation and Return on Investment over a Period of 10 Years—All Depreciation Taken During the Last Year and 7 Per Cent Return on Depreciated Investment

(1) Year	(2) Depreciated Investment Beginning of Year	(3) Annual Depre- ciation	(4) 7% Re- turn on Column (2)	(5) Return and Depreciation Columns (3) + (4)	(6) Present Worth Factor @ 7%	(7) Present Worth Columns (5) × (6)
1	$1,000.00	$.......	$ 70.00	$ 70.00	.9345794	$ 65.42056
2	1,000.00	....	70.00	70.00	.8734387	61.14071
3	1,000.00	....	70.00	70.00	.8162979	57.14085
4	1,000.00	....	70.00	70.00	.7628952	53.40266
5	1,000.00	....	70.00	70.00	.7129862	49.90903
6	1,000.00	....	70.00	70.00	.6663422	46.64395
7	1,000.00	....	70.00	70.00	.6227497	43.59248
8	1,000.00	....	70.00	70.00	.5820091	40.74064
9	1,000.00	....	70.00	70.00	.5439337	38.07536
10	1,000.00	1,000.00	70.00	1,070.00	.5083493	543.93375
		$1,000.00	$700.00	$1,700.00		$999.99999

MATHEMATICAL DEMONSTRATION OF THEOREM THAT THE PRESENT WORTH OF REPAYMENTS OF PORTIONS OF A CAPITAL SUM PLUS RETURN ON UNREPAID PORTION OF SUCH CAPITAL SUM IS EQUAL TO THE CAPITAL SUM

Let A = a capital sum
r = rate of return
L = term of years over which repayment is made
n = a number of years

I. Repayment on Straight Line Basis, Return on Cost Less Repayment

Year	*Repayment plus Return in Year*	*Present Worth of Repayment plus Return*
1	$rA + \frac{A}{L}$	$\left[r(A) + \frac{A}{L}\right]\frac{1}{1+r}$
2	$r\left(A - \frac{A}{L}\right) + \frac{A}{L}$	$\left[r\left(A - \frac{A}{L}\right) + \frac{A}{L}\right]\frac{1}{(1+r)^2}$
.		
.		
.		
.		
L	$r\left[A - \frac{(L-1)A}{L}\right] + \frac{A}{L}$	$\left[r\left\{A - \frac{(L-1)A}{L}\right\} + \frac{A}{L}\right]\frac{1}{(1+r)^L}$

Let S = Sum of present worths of repayments plus return

$$S = A\left[\frac{Lr+1}{L}\left\{\frac{1}{1+r}+\frac{1}{(1+r)^2}+\cdots+\frac{1}{(1+r)^L}\right\} - \left\{\sum_{n=1}^{n=L}\frac{(n-1)r}{L}\cdot\frac{1}{(1+r)^n}\right\}\right]$$

$$= A\left[\frac{Lr+1}{L}\left\{\frac{(1+r)^L-1}{r(1+r)^L}\right\} - \frac{1}{L}\left\{\frac{(1+r)^L-1}{r(1+r)^L} - \frac{L}{(1+r)^L}\right\}\right]$$

$$= A\left[\frac{Lr+1}{L}\left\{\frac{(1+r)^L-1}{r(1+r)^L}\right\} - \frac{1}{L}\left\{\frac{(1+r)^L-1-Lr}{r(1+r)^L}\right\}\right]$$

$$= A\left[1 - \frac{1}{(1+r)^L} + \frac{(1+r)^L-1}{Lr(1+r)^L} - \frac{(1+r)^L-1}{Lr(1+r)^L} + \frac{1}{(1+r)^L}\right]$$

$$= A$$

II. Repayment on Sinking Fund Basis, Return on Cost

Year	*Repayment plus Return in Year*	*Present Worth of Repayment plus Return*
1	$\frac{rA}{(1+r)^L-1}+rA$	$\left[\frac{rA}{(1+r)^L-1}+rA\right]\frac{1}{1+r}$
2	$\frac{rA}{(1+r)^L-1}+rA$	$\left[\frac{rA}{(1+r)^L-1}+rA\right]\frac{1}{(1+r)^2}$
3	$\frac{rA}{(1+r)^L-1}+rA$	$\left[\frac{rA}{(1+r)^L-1}+rA\right]\frac{1}{(1+r)^3}$
.		
.		
.		
L	$\frac{rA}{(1+r)^L-1}+rA$	$\left[\frac{rA}{(1+r)^L-1}+rA\right]\frac{1}{(1+r)^L}$

$$S = \left[\frac{rA}{(1+r)^L-1}+rA\right]\left[\frac{1}{1+r}+\frac{1}{(1+r)^2}+\frac{1}{(1+r)^3}+\cdots+\frac{1}{(1+r)^L}\right]$$

$$= \left[\frac{rA}{(1+r)^L-1}+rA\right]\left[\frac{(1+r)^L-1}{r(1+r)^L}\right]$$

$$= r\left[\frac{A+A(1+r)^L-A}{(1+r)^L-1}\right]\left[\frac{(1+r)^L-1}{r(1+r)^L}\right] = A$$

III. Repayment on Sinking Fund Basis, Return on Cost Less Repayment

Year	Repayment plus Return in Year	Present Worth of Repayment plus Return
1	$\frac{rA}{(1+r)^L-1}+rA$	$\left[\frac{rA}{(1+r)^L-1}+rA\right]\frac{1}{1+r}$
2	$\frac{rA}{(1+r)^L-1}+r\left\{A-\frac{rA}{(1+r)^L-1}\right\}$	$\left[\frac{rA}{(1+r)^L-1}+r\left\{A-\frac{rA}{(1+r)^L-1}\right\}\right]\frac{1}{(1+r)^2}$
.		
.		
.		
$L-1$	$\frac{rA}{(1+r)^L-1}+r\left\{A-\frac{(L-2)rA}{(1+r)^L-1}\right\}$	$\left[\frac{rA}{(1+r)^L-1}+r\left\{A-\frac{(L-2)rA}{(1+r)^L-1}\right\}\right]\frac{1}{(1+r)^{L-1}}$
L	$\frac{rA}{(1+r)^L-1}+r\left\{A-\frac{(L-1)rA}{(1+r)^L-1}\right\}+\left\{A-\frac{LrA}{(1+r)^L-1}\right\}$	$\left[\frac{rA}{(1+r)^L-1}+r\left\{A-\frac{(L-1)rA}{(1+r)^L-1}\right\}+\left\{A-\frac{LrA}{(1+r)^L-1}\right\}\right]\frac{1}{(1+r)^L}$

$$S = A\left[\left\{\frac{r}{(1+r)^L-1}\right\}\left\{\frac{(1+r)^L-1}{r(1+r)^L}\right\}+r\left\{\frac{(1+r)^L-1}{r(1+r)^L}\right\} - \frac{r}{(1+r)^L-1}\left\{\frac{(1+r)^L-1}{r(1+r)^L}-\frac{L}{(1+r)^L}\right\}+\frac{1}{(1+r)^L}\left\{1-\frac{Lr}{(1+r)^L-1}\right\}\right]$$

$$= A\left[\frac{1}{(1+r)^L}+1-\frac{1}{(1+r)^L}-\frac{1}{(1+r)^L}+\frac{Lr}{(1+r)^L}\left\{\frac{1}{(1+r)^L-1}\right\}+\frac{1}{(1+r)^L}\left\{1-\frac{Lr}{(1+r)^L-1}\right\}\right]$$

$$= A\left[1-\frac{1}{(1+r)^L}+\frac{Lr}{(1+r)^L}\left\{\frac{1}{(1+r)^L-1}\right\}+\frac{1}{(1+r)^L}-\frac{1}{(1+r)^L}\left\{\frac{Lr}{(1+r)^L-1}\right\}\right]=A$$

IV. Full Repayment at Retirement, Return on Cost

Year	Repayment plus Return in Year	Present Worth of Repayment plus Return
1	rA	$rA\dfrac{1}{1+r}$
2	rA	$rA\dfrac{1}{(1+r)^2}$
3	rA	$rA\dfrac{1}{(1+r)^3}$
.		
.		
.		
.		
L	$rA + A$	$\left[rA + A\right]\dfrac{1}{(1+r)^L}$

$$S = rA\left[\frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdot + \frac{1}{(1+r)^L}\right] + A\,\frac{1}{(1+r)^L}$$

$$= rA\left[\frac{(1+r)^L - 1}{r(1+r)^L}\right] + A\,\frac{1}{(1+r)^L}$$

$$= A\left[\frac{(1+r)^L - 1 + 1}{(1+r)^L}\right] = A$$

V. Repayment Irregular, Return on Cost Less Repayment

Let $A_1, A_2, \cdots A_n$ be repayments in a year

Year	*Repayment plus Return in Year*	*Present Worth of Repayment plus Return*
1	$A_1 + rA$	$\left[A_1 + rA\right]\frac{1}{1+r}$
2	$A_2 + r(A - A_1)$	$\left[A_2 + r(A - A_1)\right]\frac{1}{(1+r)^2}$
.		
.		
n	$A_n + r\left[A - (A_1 + A_2 + \cdots + A_{n-1})\right]$	$\left[A_n + r\left\{A - (A_1 + A_2 + \cdots + A_{n-1})\right\}\right]\frac{1}{(1+r)^n}$
.		
.		
L	$A_L + r\left[A - (A_1 + A_2 + \cdots + A_{L-1})\right]$	$\left[A_L + r\left\{A - (A_1 + A_2 + \cdot \quad + A_{L-1})\right\}\right]\frac{1}{(1+r)^L}$

$$S = rA\left[\frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^L}\right] + \sum_{n=1}^{L}\left[\frac{A_n - r(A_{n-1})}{(1+r)^n}\right]$$

$$= rA\left[\frac{(1+r)^L - 1}{r(1+r)^L}\right] + \left[\frac{A_1}{1+r} + \frac{A_2}{(1+r)^2} + \cdots + \frac{A_L}{(1+r)^L}\right] - r\left[\frac{(A_1 + A_2 + \cdots + A_{L-1})}{(1+r)^L}\right]$$

$$- r\left[\frac{(A_1 + A_2 + \cdots + A_{L-2})}{(1+r)^{L-1}}\right] - \cdots - \frac{rA_1}{(1+r)^2}$$

$$= A - \frac{A}{(1+r)^L} + \left[\frac{A_1}{1+r} + \frac{A_2}{(1+r)^2} + \cdots + \frac{A_L}{(1+r)^L}\right]$$

$$- rA_1\left[\frac{(1+r)^{L-1} - 1}{r(1+r)^L}\right] - rA_2\left[\frac{(1+r)^{L-2} - 1}{r(1+r)^L}\right] \cdots$$

$$= A - \frac{A}{(1+r)^L} + \frac{A_1[(1+r)^{L-1} - (1+r)^{L-1} + 1]}{(1+r)^L} + \frac{A_2[(1+r)^{L-2} - (1+r)^{L-2} + 1]}{(1+r)^L} + \cdots$$

$$= A - \frac{A}{(1+r)^L} + \frac{1}{(1+r)^L}(A_1 + A_2 + \cdots + A_L)$$

but $A_1 + A_2 + \cdots + A_L = A$. Therefore $S = A - \dfrac{A}{(1+r)^L} + \dfrac{A}{(1+r)^L} = A$

CHAPTER V

COMPARISONS OF CERTAIN ASPECTS OF THE PUBLIC UTILITY BUSINESS WITH COMPETITIVE STANDARDS

Commenting on the operation of competitive enterprises under the impingement of improvements in the art, Veblen[16] has said:

> Each new venture or extension goes into the competitive traffic of producing and selling any line of staple goods with a differential advantage, as against those that have gone before it, in the way of a lower scale of costs. *** The run of competitive prices is lowered; which means that at the new competitive prices, and with their output remaining on its old footing as regards expenses of production, the older establishments and processes will no longer yield returns commensurate with the old accepted capitalization.

The older plants must try to keep up to date by renovating worn machinery and by revamping entire plant layouts, in order to modernize both methods and equipment. There are limits to this process, because a plant cannot be kept up to date in every respect. There is, for example, the fact of irreplaceable wear and the impracticability of complete modernization short of total retirement and rebuilding. The deteriorated and outmoded plant is a relatively high cost producer moving toward the marginal position. The plant may be operated even beyond the marginal point, but its income is no longer sufficient to cover both supplementary and prime costs and it must eventually yield the field to the more modern and more economical producers.

In regulated industries, where the free market and perfect competition are completely lacking, the problems in respect of deterioration and obsoleteness are somewhat different. It is true, on the other hand, that public utilities are not perfect monopolies. Railroads are in competition with alternative means of transportation. Gas utilities, whether supplying a natural or a manufactured product, are in competition with the electric power business in certain markets. Electric utilities are in competition with isolated private generating facilities and with public power projects. Beyond this limited competition the electric power industry has a virtual monopoly. This is in the field of supplying the needs of the small commercial and domestic users.[17]

Representatives of the industry would probably point to the increased

domestic consumption with decreased rates to demonstrate that, except for minimum requirements, there is competition in supplying the consumers' demands for refrigeration, heat, and other convenient services. There might be some merit to this view but for the fact that in these areas of activity the suppliers of alternative services, such as gas, do not universally act independently. In general, real competition could be obtained only at those prices which might call into use kerosene lamps and stoves, candles, and similar substitutes. That is, if there were no regulation, prices for certain electric services would tend to rise to higher levels than are experienced under regulation. The tendency in all private enterprises is to maximize earnings and frequently this could be accomplished with higher rates and lower consumption.

If regulation were to attempt to simulate the effects of competition in every respect, the use of any rate base would be difficult, if not impossible. It might be more accurate to say that regulation attempts to limit monopoly return to a proportion of a determined capital sum for a short run. Regulation is a substitute for competition, without a doubt, but for guidance it must look to the probable influences of competition under certain visualized but unrealized conditions.

Considerations of competition disregarding other factors would cause regulation to fix prices from time to time without constant reference to original cost or "fair value." Competitive prices are determined by costs of production and the forces of consumer demands, neither of which are related to original cost of the property in service. In fact the prices are "value" determining rather than "value" determined.

In a case before the New York Commission[18] a manufactured gas company was pleading for the establishment of a "fair value" or rate base. The company was charging 80¢ per thousand cubic feet of gas when a natural gas company began to supply certain parts of the city with gas at 27½¢ per thousand. In order to try to meet this competition wherever it existed, the manufactured gas company lowered its rate to 50¢ to those customers who were located where they could make a choice between natural and manufactured gas.

The engineer for the manufactured gas company stated:

> A division of the territory between rival concerns is impracticable for many reasons, and the two courses left open for the present are the acquisition of the (natural gas) company by the (manufactured gas) company on some reasonable terms, or a war of competition which shall end in the survival of the fittest.

The Commission's comment on this view was:

> How the much dreaded war would affect the value of the (manufactured gas) company, except by impairing its earnings, he does not explain.

It is not clear at this point in the opinion, that in this situation value was of necessity affected by the competitive influences on earnings. However, at a later part, the Commission stated its understanding of the problem in these terms:

> If cost of reproduction new is (the basis of reasonable value), it leaves out of view entirely the effect of competition of natural gas and of electric light. *** Now, no person will contend for a moment that the existence of natural gas in (the city) does not seriously and to a very large extent affect the value of the property of the (manufactured gas) company.

What "fair value" is, or what the corresponding rate base should be, has, in specific cases, been determined by courts. No general formula has been advanced, but whatever may have been intended by the term "fair value," it has been made clear that accrued depreciation must be taken into account.[19] That fact indicates an attempt to simulate some of the effects of competition. The competitive business must adjust price in response to the influence of advancing art and the lower costs of newer producers. Proper accounting for depreciation would result in reduction in rate bases in regulated enterprises with ultimate reductions in rates.

Some of the causes of depreciation are factors which, in competitive enterprises, would influence prices. Deterioration is frequently followed by increased operating expenses compared with conditions when new; advancement in the art may give a newer producer an opportunity to make a better product at a lower cost; changes in the market may permit a new producer to locate more favorably and more economically than the older one. Unregulated monopoly need not take these competitive factors into account. Regulated monopolies must be governed by these considerations, not that they actually do influence prices in all areas, but that they would, were it not for the monopoly situation.

For example, modernization of existing facilities, retirement of outmoded ones and construction of up to date plants are frequently delayed by managements long after the time when, under competitive circumstances, these would have been compelled. Frequently the failure to retire outmoded equipment and plants is explained on the ground that these facilities are required for emergency or stand-by service. Often equipment is held in service because the reserves are inadequate.[20]

The New York Public Service Commission at this time is requiring the establishment of continuing property records and the retirement of property not used and useful in the public service.[21] When this state-wide program has been fulfilled studies may be made to show how much equipment formerly carried in service should be retired because it is no longer used and useful. Equipment so classified has, of course, experienced the full effects of depreciation.

The effects of deterioration and of progress in the arts do not occur uniformly or in accordance with some function of time although they do occur in a time continuum. In other words, a particular plant in a competitive field is projected toward the position of a marginal producer irregularly with respect to time. Regulation should, therefore, recognize this movement which, although it may not actually occur in the public utility field, would nevertheless take place if the full influence of competition were permitted to act. This feature of regulation would require that a loss in value of a plant be recognized as occurring when the facts as to deterioration and changes in the art would act under competitive conditions. Value as here used is that number of dollars representing the sum of the present worths of the expected gross incomes less all operating expenses (except depreciation) where the expectation of income is founded on a thorough familiarity with the business and its relations to its competitors. As the art progresses and deterioration effects a lowering of productive efficiency or a lowering of the market price, the business tends to move toward the marginal producer's position and its total incomes are decreased. Thus it is that those incomes which were previously expected are now incapable of being realized. Consequently the measure of value is reduced.

Regulation should attempt to discover those effects attributable to deterioration and progress in the art which in a competitive industry would cause a loss in value.

The discussion in earlier chapters compared the effects of the various methods of accounting for depreciation. The point was made that equity between consumers as a group and investors as a group was met under *any* consistent treatment of depreciation. Now, if we introduce the requirement that property value meet the criterion of equality with the sum of the present worths of incomes, less operating expenses (except depreciation), then the accrued depreciation can be made to reflect to some extent the loss in value which has occurred due to the effects of changes in the art, changes in demand and of deterioration. Reverting to the discussion in the earlier chapters, it does not seem reasonable to

assume that a property has lost value to the extent indicated by any life-age computation, if the facts of the case indicate that the expected incomes are such as to leave the property as a low cost producer, even though half of its predicted life is gone; or, on the other hand, that it has lost only half of its value when in fact it has declined below the position of a marginal producer. The argument in favor of estimated actual depreciation is that, as the causes of depreciation actually would effect a shift from the margin if the utility were operating in a competitive field, account is taken of these facts as they actually occur. All other methods are more or less arbitrary in creating amortization reserves not directly related to the facts as they occur.

CHAPTER VI

ESTIMATED "ACTUAL" DEPRECIATION

We proceed now to a description of the technique or method of determining estimated actual depreciation. The factors which operate together as determinants of depreciation are:

1. Estimated deterioration or wear and tear.
2. Recognized obsoleteness.
3. Demonstrated inadequacy.
4. Requirements of public authority.
5. Decline in use or demand.

Deterioration or Wear and Tear

Deterioration is the undesirable change in weight, in physical and chemical composition, in shape, or in dimensions, caused by the action of the elements or due to use or abuse. Deterioration may operate so as to make restoration partially or completely impossible. Thus deterioration may be classified into two categories, one in which restoration is possible through maintenance expenditures, the other in which maintenance is incapable of effecting any restoration. However, this distinction is not essential to our discussion until we come to the consideration of annual depreciation charges in a later chapter.

The condition of property with respect to wear and tear or deterioration may be estimated by men thoroughly familiar with the various types of equipment. In such estimating, the effects of all other factors of depreciation are omitted.

The establishment of a factor representing the deteriorated condition is based on the visualization of the property when new and properly installed, and when deteriorated to a condition requiring replacement. These subjective scale limits are such that 100% is assigned to new conditions or where there is no deterioration; and 0% is assigned to that condition at which there is no further use value, or where there is complete deterioration. The actual condition of the property is assigned a factor which represents the inspector's judgment of where this condition stands on a scale running from 100% to 0%.

The application of such judgments involves a twofold process; one,

in which there is judgment in respect of physical condition as an operating or engineering factor; the other, in which there is a judgment of value as affected by this condition.

The physical condition is interpreted as a proportion of the cost new representing the remaining value on the assumption that the asset in question has suffered no diminution in respect of value other than that due to wear and tear.

It is not to be expected that an inspection alone will furnish a sound basis for judging the physical condition. A study of the statistical and operating records, which throws light on the present operating efficiency compared with the original situation when the property was new, will, in combination with the visual inspection, form the basis for a final judgment.

Some types of property lend themselves more readily to objective tests as to the extent of physical deterioration, and it should be emphasized that the desideratum is objectivity. Until objective tests have been devised for all types of property, subjective judgments must be relied upon.

The rotting of wood poles is a type of deterioration which may be analyzed by certain objective tests. The interpretation of these, however, rests on judgment. A rotting wood pole may at each stage in its decay be considered to be worth the same as a new pole the dimensions of which are equal to those of the remaining sound wood.

The development of a formulation applicable to this method is given at the end of this chapter. (Page 71.)

If the strength of a pole when new is represented by a physical condition factor of 100% and if, when the strength is zero the physical condition factor is 0%, the result is a scale for measuring physical condition at any ground line diameter which is applicable to the remaining sound wood.

This method of measuring physical condition may be labelled the "method of actual strength variation." Under actual loading conditions there is a minimum diameter consistent with safe practice, below which a pole should be removed. If in the face of this requirement, the pole nevertheless is left standing, it is postulated that the physical condition drops to zero at the threshold set by engineering safety standards. In this connection it should be noted that accrued depreciation in excess of the total original cost of the pole is meaningless.

If, as above, the physical condition of a new pole is represented by a condition factor of 100%, but the 0% condition factor is established at that strength which corresponds to the safe removal diameter rather

than to zero diameter, the same objectivity in testing obtains, but the interpretation of the results is somewhat different. This method for measuring physical condition may be labelled the "method of remaining useful strength variation."

The study of physical condition as measured on a scale established by substituting costs of new poles, first requires the determination of the costs of substitute poles such that, for a given type of wood and constant height, these costs may be arranged as a function of ground line diameter. If the cost of each "pole diameter" is associated with respective pole strengths, and if at a diameter corresponding to the original pole diameter the value is set at 100% whereas at a diameter of zero the value is set at 0%, the result is a new scale based on values varying with remaining diameter. For purposes of identification this method for measuring physical condition is labelled the "method of value variation based on actual diameter." Below the safe removal diameter, the value drops to zero.

If the value of a pole at safe removal diameter is fixed at 0% and the value at original diameter is represented, as before, by a factor of 100%, then there is a new scale for measuring physical condition. For reference purposes this method is labelled the "method of value variation based on remaining useful diameter."

The accompanying diagram illustrates the four methods as applied to any pole having an original ground line diameter of 12″ and a safe engineering removal diameter of 6″.

The application of these methods to a public utility company actually owning approximately 30,000 poles and based on examinations of a random sample of approximately 3,000 poles follows:

	Number of Poles	Method	Condition Factor
No rot	2173	Useful strength	100%
		Actual "	100%
		Useful value	100%
		Actual "	100%
Rotted	809	Useful strength	56.3%
		Actual "	67.5%
		Useful value	61.0%
		Actual "	76.4%
Rotted below safe		Useful strength	0%
removal diameter	40	Actual "	0%
		Useful value	0%
		Actual "	0%
	3022		

Weighting—Useful Strength Method

$$
\begin{array}{rcl}
2173 \times 100\% & = & 2173.000 \\
809 \times 56.3\% & = & 455.467 \\
40 \times 0\% & = & 0 \\
\hline
3022 & & 2628.467 \\
2628 \div 3022 & = & 87\%
\end{array}
$$

Weighting—Actual Strength Method

$$
\begin{array}{rcl}
2173 \times 100\% & = & 2173.000 \\
809 \times 67.5\% & = & 526.075 \\
40 \times 0\% & = & 0 \\
\hline
3022 & & 2699.075 \\
2699 \div 3022 & = & 89\%
\end{array}
$$

Weighting—Value Based on Remaining Useful Diameter

$$
\begin{array}{rcl}
2173 \times 100\% & = & 2173.000 \\
809 \times 61\% & = & 493.490 \\
40 \times 0\% & = & 0 \\
\hline
3022 & & 2666.490 \\
2666 \div 3022 & = & 88\%
\end{array}
$$

Weighting—Value Based on Actual Diameter

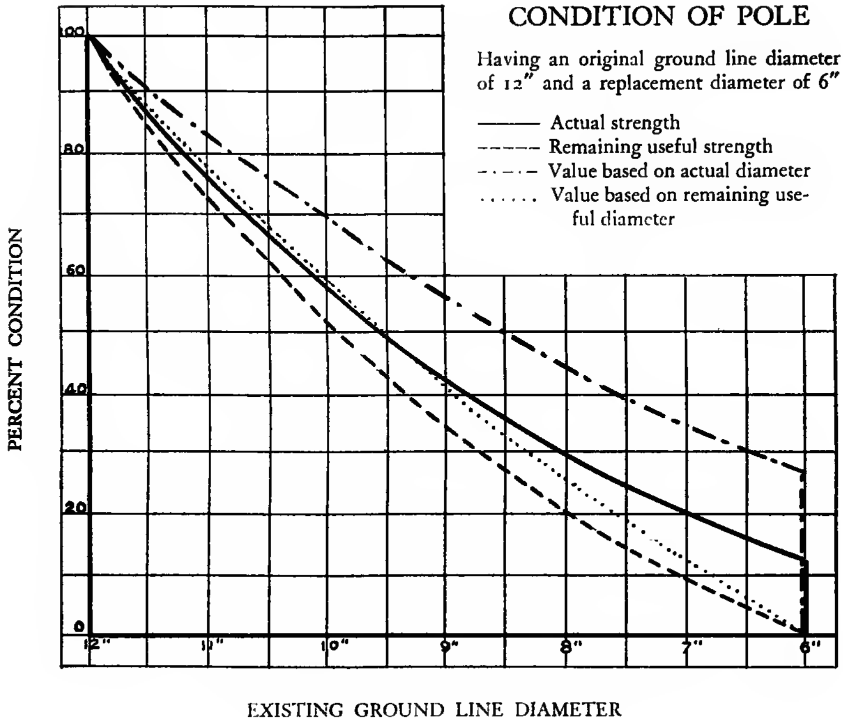
$$
\begin{array}{rcl}
2173 \times 100\% & = & 2173.000 \\
809 \times 76.4\% & = & 618.076 \\
40 \times 0\% & = & 0 \\
\hline
3022 & & 2791.076 \\
2791 \div 3022 & = & 92.2\%
\end{array}
$$

Weighted Conditions

Useful strength	87%
Actual strength	89%
Value based on remaining diameter	88%
Value based on actual diameter	92%

There is a spread between the most divergent methods which is not too great to make it reasonable to utilize any one of them as a means of measuring condition as to physical deterioration.

It seems, at this time, as if the method of actual strength determination would yield the most reasonable and consistent results. It has the advantage that it is not affected by changes in wire loading excepting as to the location of the cut-off point at the safe diameter. Furthermore, equal extents of rot in poles of similar dimensions and of like kind result in the same percent condition.



It must be admitted that there are few types of property for which objective tests have been devised. Until such time as tests are developed for all types of property efforts should be continued toward their invention. In passing, it may be worth commenting that in judging human values such as army promotions, school system promotions, civil service tenure, etc., judgments are relied upon using standards for each characteristic or attribute as represented in the examiner's mind by a series of paragons.

Those types of property for which no objective tests are available are assigned condition factors based on judgment alone. The greatest care must be taken in training inspectors so as to eliminate those judgments which are based on bias, on lack of experience or on ignorance. The accompanying tables illustrate the application of the method to 40 distribution substations and several generating stations respectively. Each station was examined and each piece of equipment inspected. The operating records as to maintenance and performance were studied and

operating crews were questioned. The judgments fixing physical condition were made independently and the results tabulated. The widest spreads between the inspectors should be noted in the case of the judgments of inspector H. M. Z. Close questioning of this inspector revealed very little understanding of what was expected and although he was well qualified in many ways, certain major types of electrical equipment were foreign to him. The persistence of the judgment of 99% was due to H. M. Z.'s failure to take into account any factors save superficial and surface appearance. Based on this experience great care was taken in reviewing the bases for judgments.

Subsequently 45 substations belonging to another large electric utility were inspected and the results are set forth in the third table.

The tendency of subjective estimates of condition, by different experts working independently, to approximate each other is significant. The "value" of real estate, of automobiles, of goods in general is fixed by the aggregate operations of many opinions. The loss in "value" due to the effects of deterioration, in the absence of accepted objective standards, can properly be approximated by the opinions of trained experts, who have themselves been tested to determine their ability to pass judgments within small deviations from the mean of many expert judgments.

The scale of such judgments must be no more precise than the error of the mean. After years of experience it appears advisable to adopt the following:

Condition new	100%
	99
	98
	97
	96
	95
	94
	93
	92
	91
	90
	85
	80
	75
	70
	60
	50
	40
	20
Condition requiring replacement or withdrawal from service	0

Steam-Electric Generating Stations—Physical Condition Percentages as of January, 1937

Station	Year in Operation	Building	Boiler Plant HMZ	Boiler Plant FJL	Boiler Plant JB	Boiler Plant TB	Turbo Generators HMZ	Turbo Generators FJL	Turbo Generators JB	Turbo Generators TB	Electric Plant RJH	Electric Plant FJL	Electric Plant JB	Electric Plant TB
A	1926	92	97		93	95	99		93	95	99		98	97
B	1921	90	97	75	85	75	98	85	85	85	98	95	97	95
C	1901, 1905	87.3		60	60	50		85	75	80		90	90	90
D	1913	90	95	80	75	75	98	90	90	90	93	95	95	95
E	About 1907		91				98				97			
F	" 1907	85	90				90				95			
G	1924	93	93		90	92.5	98		95	95	99		98	98
H	1897	80	78	65		60	97	90		90	97	95		
I	1904	80	93	55		50	96	85		90	97	90		
J	1926	80												

When estimating a physical condition there is a lack of an articulate theory of transition from the judgment of the per cent of physical condition to the judgment of value. We know that there is a correlation of some kind and the goal is to discover its nature.

Electric Distribution Substations—Physical Condition Percentages as of January, 1937

		Condition Percentages Designation of Inspectors		
Station	Date in Service	FJL	HMZ	WHC
No. 1	1923	97	99	96
2	1906	90	99	93
3	1921	90	99	90
4	1899	85	99	90
5	1924	90	99	95
6	1904	85	99	90
7	1920	95	98	96
8	1903	95	99	95
9	1927	98	99	98
10	1921	95	99	95
11	1902	85	99	90
12	1908	90	99	90
13	1906	95	99	90
14	1903		99	
15	1902		99	
16	1907		99	
17	1925		99	
18	1901		99	
19	1913	90	99	95
20	1904	85	99	90
21	1922	95	99	95
22	1925	95	99	95
23	1915	95	99	99
24	1904	95		96
25	1926	98		97
26	1910	85		93
27	1925	97	99	97
28	1923	95		97
29	1907	90	99	95
30	1927	98		98
31	1906	95		97
32	1907	95		96
33	1904	95		97
34	1908	98		96
35	1900	95		97
36	1925	98		98
37	1902	95		93
38	1902	95		97
39	1899	95		95
40	1907	90		96

Electrical Substations—Physical Condition Percentages as of January, 1938

Station	Date in Service	Condition Percentages Designation of Inspectors FJL	WHC	RJH
No. 1	1925	98	98	98
2	1917	90	92	96
3	1922	75	85	85
4	1925	98	98	96
5	1925	95	96	96
6	1925	80	80	98
7	1928	98	98	98
8	1900	95	95	96
9	1929	98	98	97
10	1926	85	90	95
11	1919	90	85	92
12	1920	98	95	98
13	1925	95	97	96
14	1918	95	95	96
15	1929	99	98	98
16	1918	98	98	98
17	1926	95	95	97
18	1921	97	95	96
19	1937	99	98	98
20	1936	99	99	98
21	1935	99	99	99
22	1925	95	98	97
23		95	97	98
24		98	97	97
25		95	96	97
26		98	98	98
27		95	95	97
28		95	94	97
29	1922	60	60	65
30		90	93	94
31		98	99	97
32		90	93	94
33	1937	80	85	85
34	1925	80	85	94
35	1936	100	100	99
36		95	97	97
37		95	96	98
38		85	85	90
39		90	90	90
40		99	97	95
41		95	95	98
42		90	93	95
43		95	95	96
44		95	95	98
45		96	98	97

Obsoleteness

This term means the loss in value which has occurred in a property due to the fact that there is actually available a substitute plan, design, scheme or product which would operate in the place of the existing property so as to bring about increased safety, improved reliability, or the creation of a new supply-price curve. The substitute would be able to supply a given quantity of product at an overall cost lower than that at which the existing property can produce the same quantity of product.

It may be recalled that loss in value arises out of that phase of the situation which influences the expected future earnings of a property with the advent of a lower cost producer.

The loss in value due to obsoleteness has no meaning for utility enterprises apart from that which is injected by considerations of competitive price standards. Assuming that the utility enjoys an unrestricted monopoly in every sense, there is little incentive to reduce operating costs by introducing improved methods or equipment, for such cost reduction is not reflected in increased net earnings. If the net earnings had previously been up to the regulated limit, then any increase of net earnings must result in the regulatory commission requiring a reduction of rates. The constant introduction into utility systems of improved methods and designs indicates the pressure of competition. The emulation of competition in studying obsoleteness need not be perfect in order to be of some practical utility.

Attention is called to the use of the term "obsoleteness," in preference to the usual accounting term "obsolescence." The thought which the word is intended to convey is the extent to which loss in value has occurred owing to progress in the art without reference to possible future developments, information concerning which is not yet available.*

From all of this it seems reasonable to utilize the concept of the degree of obsoleteness as descriptive of an existing condition such that 100% obsoleteness indicates that situation in which a property has become completely obsolete, in that it has lost 100% of its value. Obsolescence, on the other hand, connotes that dynamic situation in which a

* The choice of a suitable word presents one with a dilemma based on the absence of an unimpeachable word in the English language connoting the precise meaning. By definition of almost universal acceptance "obsoleteness" means "complete obsoleteness" and hence "partial obsoleteness" becomes a contradiction in terms. On the other hand "obsolescence" connotes a process of change rather than a status. Hence either word is wrong; and, unless a new word were coined one is forced to choose whichever *poor* word one believes will give rise to the least error. For this reason the word "obsoleteness" will be used as indicated in the main text of the paper.

property is moving from one condition to another on its way toward complete desuetude or loss in value. For our purpose here, it is not necessary to predict the future or to guess that new and yet unknown developments or progress will cause the value of this property under consideration to fall. It is only necessary to recognize the existing facts which cause loss or have caused loss in value, that is, a decline in previously expected future earnings.

Throughout this discussion the loss in value has been measured along a scale running from 100%, or condition when new, to 0%, or condition when totally deteriorated or wholly obsolete. This does not give any consideration to salvage or scrap value. The view taken with respect to scrap value is that it should not be considered in determining the accrued loss in value for rate making purposes. The value of property is that which is associated with its use; scrap value is that associated with a realization of money after the property has been withdrawn from service.

Property may be worthless as a part of an operating utility and yet have some scrap value upon being withdrawn from service. In computing a rate base the scrap value should not be considered. However, in requiring rate payers to reimburse owners for property through annual depreciation charges and in setting up a reserve for depreciation, scrap value must be considered.

In other words, the rate payers should be called upon to reimburse only that portion of the total loss in value which is not covered by scrap value. The depreciation reserve is built up from annual depreciation credits so that under this scheme, the depreciation reserve does not reflect the total loss in value. The computation of a rate base, utilizing the depreciation reserve as a measure of loss in value is, under these conditions, erroneous. It might be better to set up the depreciation reserve so as to reflect the lotal loss in value, earmarking a portion thereof as eventually to be credited from scrap value realization. This would permit the reserve to be a measure of loss in value and yet permit the annual charges for depreciation to be adjusted for the off-setting effects of scrap value realizations. This will be discussed further in the chapter on annual depreciation.

The elements of safety and reliability are frequently incapable of being studied objectively, and the conclusions as to their effects upon loss in value are necessarily based on subjective reasons. The element of decreased relative costs, on the other hand, is more often than not a matter for reasonable economic analysis which is mainly objective.

While at this time all the refinements of an objective test have not been explored, there is a test which is suggested as a preliminary guide to the determination of the extent of obsoleteness. The value of an existing property is the sum of the present worths of the expected incomes less all operating expenses, excluding depreciation. In a previous chapter, it has been stated that the income of a property in a competitive market must be sufficient to cover the operating expenses, the depreciation charges, and a return on capital remaining in the business. This is the difference between the original investment and the accrued depreciation. The development of the mathematical basis for this statement is given in the appendix.* The equation for the determination of the existence and extent of obsoleteness is

$$\frac{C - C_e + \sum_{i=0}^{N} pw[O_i - (O_e)_i]}{C} \times 100$$

The substitution of actual figures for the symbols yields a result which is to be interpreted as follows. A negative sign indicates that the substitution of a modern plant for the existing one would effect no overall savings. A positive sign indicates that some overall savings would be effected by such substitution, but the magnitude of the savings might be insufficient to justify the retirement of the existing plant. In other words, there might be some obsoleteness which, although recognizable, would not make the existing facilities completely obsolete.

Obsoleteness must be measured under conditions of operation for which the property was designed and installed. For example, a machine designed to carry a specific load for a given number of hours a year, or to produce a given number of units of product per year, becomes obsolete to a degree measured by the substitution of a lower cost producer, performing the same duties. The argument that, due to the fact that a particular machine has been relieved of its duties or has been relegated to emergency operations, there is no obsoleteness because there are no possible operating savings, must fail. The particular machine was bought to perform its original duties and would not have been bought for emergency service. It is possible that if some machine were required for emergency service it would be a less costly, or smaller, or different type one. The very fact that the original machine has been relegated to reduced service duties is a recognition of decline in value.

* P. 71, Appendix Part B.

An automobile, a typewriter, a desk, a generating station are all obsolete to the extent that the art has progressed; and this is true whether or not the property in question is heavily or lightly used. As far as obsoleteness is concerned, the basis for computing it is the use for which it was intended, i.e. the "rated" load is the starting datum.

To illustrate the application of the formula, and to explore the difficulties in determining obsoleteness, an example will be presented, involving the electric generating facilities of a large power system.

Obsoleteness occurs in a system of generating stations when more economical plants become available as possible substitutes. The determination of the extent of obsoleteness depends upon the comparison of the overall costs of energy for the existing system with those for the system of modern plants. The operation of the present plants might be studied as an integrated system or unit, controlled in such a way as to produce energy at the lowest cost of which this system is capable. On the other hand, the present plants might be studied as individuals, without considering their interdependent operation. The costs referred to are those which apply to the system as a whole when operating as a unit and so the obsoleteness is applicable to the system as a whole. It is suggested that if it is necessary to allocate the system obsoleteness to the several plants and parts thereof, then the study of the individual plants will furnish a guide to such allocation.

The example deals with a system distributed as follows and for which a modern substitute is as noted.

	Existing System			Substitute System		
Station	Initial Date in Service	Steam Pressure lbs. Gauge	Gross Capacity Kilowatts	Steam Pressure lbs. Gauge	Degrees F. Steam Temp.	Gross Capacity Kilowatts
A	1920	265 to 586	262,500	1,200	900	270,000
B	1930	375	60,000	1,200	900	185,000
C	1914	175	116,500			
D	1927	200	5,000	1,200	900	5,000
			444,000			460,000

The annual output of the system is 1,844,000,000 kilowatt-hours, with a 15 minute peak of 364,000 kilowatts and a load factor of 57.9%. The substitute plants have a heat rate of about 11,500 British Thermal Units per kilowatt-hour of station send-out, allowing for standby and "peak prepared for" losses, and considering a plant capacity factor of 40% or

better. Below 40% capacity factor, the heat rate rises rapidly, and at 10 to 15% the heat rate is about 14,500 British Thermal Units per kilowatt-hour.

Coal at station "A" costs about $2.02 per ton, whereas at the other stations it costs about $2.72 per ton, corresponding to costs of 7.77¢ and 10.48¢ per million British Thermal Units for coal testing 13,000 BTU per pound. The annual fuel costs of the substitute system are determined as follows:

Station	Annual KW Hr Output	Capacity Factor	Heat Rate BTU Per KWH	Coal ¢ per Million BTU	Annual Fuel Cost
A	1,630,000,000	73%	11,500	7.77	$1,460,000
B & C	200,000,000	13%	14,500	10.48	304,000
D	14,000,000	–		10.48	
	1,844,000,000				$1,764,000

Without presenting the details of the remaining calculations, which are familiar to all power system engineers, the results of the comparison of annual costs are as follows:

	Present System	Substitute System
Annual Fuel Costs	$2,715,000	$1,764,000
Other Operating Expenses	1,514,000	1,231,000
Taxes and Insurance	552,000	552,000
Total	$4,781,000	$3,547,000
Estimated Cost of Present Construction	$47,237,000	$50,600,000

In order to substitute these values in the formula it is necessary to state the results in symbolic form.

$$C = 47{,}237{,}000$$
$$C_e = 50{,}600{,}000$$
$$O = 4{,}781{,}000$$
$$O_e = 3{,}547{,}000$$

The substitute system costs $3,363,000 more than the existing system, but the annual savings are estimated to be $1,234,000. The annual savings are to be capitalized by the use of the concept of present worth previously discussed.*

* Pages 10, 58, also 71, Part B.

Each future year's savings are "worth" a certain fraction thereof today, depending upon the assumed rate of interest or return and the number of years to the future date, when the savings are realized. The rate of return falls within that range which has been established by many regulatory bodies, namely 6% to 8%. The present worth, to which reference has already been made, is that sum which, if placed at interest compounded periodically, will permit the payment of a fixed sum at uniform intervals of time.

R. H. Montgomery[22] presents this concept in mathematical terms

$$\text{Present Worth} = 1 - \frac{1}{(1+i)^n} = \frac{D}{i}$$

i = Annual rate of return or interest
n = Number of years
D = Compound interest

So that the variations in results, arising from the assumption of different lengths of time, may be clearly seen, the following table has been prepared:

Assumed Elapsed Time	Rate of Return or Interest 6%	7%	8%
	Multipliers Corresponding to Assumed Conditions		
10	7.36	7.02	6.71
15	9.71	9.11	8.56
20	11.47	10.59	9.82
25	12.78	11.65	10.67
Infinite	16.67	14.29	12.50

From the table it is seen that the assumption of the time element is of greater importance than the assumption of the rate of return.

The time assumed is not necessarily an estimate of the expected life of the property. A management, faced with the necessity for deciding whether or not to make a substantial capital outlay, must find a basis in expected fixed charges and operating expenses. The fixed charges used in such decisions are usually composed of estimates of taxes, insurance, return and amortization. The latter is based on that period of time over which the management desires to recover the original investment.*

* The matter of the time period is the most troublesome in the development of this analysis. It may be helpful to engage in a somewhat different attack on the problem as set forth below:

Just what do we mean when we say "this property is partially obsolete"? Do we not

The engineering studies of many engineering companies and of such writers as Justin & Mervine[23] and Barnard, Ellenwood & Hershfeld[24] utilize fixed charges, ranging from 10% to 15%, incorporating amortization or depreciation charges ranging from 3% to 5%. In terms of so-called straight line depreciation, this corresponds to a range of life of 33 1-3 down to 20 years. On the basis of a 5% sinking fund, the range of life is 25 to 15 years. But it is clear that these same organizations and writers do not believe that these are the expected actual lives, because

mean that a new type of property would, if installed in the place of the present property, effect certain annual savings in the algebraic sum of fixed charges and operating expenses? Assuming that the older property would live forever except for removal due to obsolescence, and that the new property would have a similar life expectancy, then the savings would be perpetual and the "present worth factor" would be the reciprocal of the interest rate. Suppose that the old property consumed fuel in the form of coal and the substitute consumed fuel in the form of oil, the same savings would then be perpetual only on the assumption that the coal and oil fuels would remain at the same price level or if they varied that the differential would be maintained. This is unreasonable and yet at the time of study we recognize obsoleteness and the management must make a decision.

What should be the basis of the decision? Shall it be based on perpetuity which is unreasonable or shall it be based on assumed or predicted lives of property? If based on assumed or predicted lives why has no cognizance been taken of probable changes in relative fuel costs?

The decision is not based on whim. It is probably founded on the following pattern of rationalization:

Were the new or substitute property to live infinitely long, and were the operating cost differentials to remain constant the savings would be perpetual. But these assumptions are invalid. If the property, both old or substitute, had a known life of 10 more years from now, and the operating cost differential remained the same, then the existing property is obsolete to such and such a degree. But we do not even know that the substitute will not be partially outmoded tomorrow or even next year and we do not know that operating cost differential will not change the day after the substitute is installed and to such an extent as to obliterate all savings. If such a change occurred the old machine would not then be obsolete to any degree. Afraid to install a substitute because differential operating costs might obliterate savings, or advancement in the art make the substitute completely obsolete within a short time, we might see potential savings go unrealized year after year waiting for a constancy or certainty which never will come. Therefore, we will assume a time period not so long as to approximate the results of perpetuity and not so short as to make us comfortable with respect to the lack of chance. We will select time periods varying between 15 and 30 years. Within the range of probable rates of return from 6% to 8% not much change in obsoleteness computation occurs in the use of time periods over twenty-five years. As a guide to the judgment of loss in value we shall use twenty-five years as a standard, recognizing that we would come to a far different conclusion if the time assumed were reduced.

K. G. Hagstroem in an article entitled "Remarks on the Theory of Depreciation" published in *Econometrica,* October, 1939, states: "When an engineer builds a bridge, he does not construct it so that the absolute minimum of material is consumed in order to obtain the solidity which is needed. He is working with wide margins; if he does not he is to be blamed. In the same way I suppose that very few of the transactions in this world are made in accordance with the ideal equation of the mathematical theory of exchange."

some advocate retirement expense accounting and others advocate age-life depreciation using lives which are inconsistent with the previous assumption.

Another view of the time assumed, is that it represents a reasonable investment period which might be more or less than the life estimated on the basis of physical deterioration, alone. It cannot be compared to "service life," for this is the period a property may be used, considering all causes of retirement of which obsoleteness is only one.

The completion of the example requires the selection of a multiplier. For present purposes it will be assumed that the time is 20 years and the rate 7% corresponding to a multiplier of 11.65.

The equation with values substituted, is as follows:

$$100 \times \frac{(47{,}237{,}000-50{,}600{,}000) + 11.65\,(4{,}781{,}000-3{,}547{,}000)}{47{,}237{,}000}$$

$$100 \times \frac{-3{,}363{,}000 + 11.65 \times 1{,}234{,}000}{47{,}237{,}000}$$

$$\text{Obsoleteness} = 23.3\%$$

It is not here contended that the formula yields anything more than a guide to judgment. For example, it throws no light on the extent of obsoleteness due to factors which might effect greater safety or reliability and which cannot be assigned money values.

Decline in Use or in Demand

This element of depreciation is concerned with the diminution of demands for the service of the property in question. As an example, we may assume eight electrical distribution substations designed to serve a demand of 312,500 KVA imposed by industrial plants, numerous commercial establishments and a large number of homes. Some time after the completion of these substations, some of the industrial plants became bankrupt and others moved away. The load fell and the best estimate of its ultimate magnitude was estimated at 204,450 KVA. This could be adequately served by substations of smaller installed capacities. The difference in costs, between stations appropriate to supply 204,450 KVA of load and the existing ones, is what is meant by depreciation due to decline in use or in demand.

Table of Substation Capacities and Loads

Sub-station	Capacity: Installed	Capacity: Operating With Allowance For Reserve	Actual Demand 1936 Peak KVA	Best Estimate of Ultimate Peak Load KVA
1	40,000 KVA	37,500 KVA	20,736	12,317
2	40,000	37,500	21,060	26,641
3	40,000	37,500	12,838	16,240
4	40,000	37,500	22,194	28,075
5	50,000	50,000*	31,823	46,462
6	40,000	37,500	20,511	29,946
7	40,000	37,500	19,608	28,628
8	40,000	37,500	19,492	28,458
	330,000 KVA	312,500 KVA	168,262	204,450

(Transformers are in units of 10,000 KVA per bank)

* All units installed are needed to carry the load.

From a study of the table it is evident that the capacities might be reduced in several stations and that one 10,000 KVA transformer and its associated equipment might be eliminated in Stations 1, 2, 3, 4, 7 and 8. In addition, certain other studies were made, which revealed the fact that some small amount of equipment in each station now serves no useful purpose, although at one time this may not have been the case.

Table Showing Differences in Cost

Sub-station	Cost of Station As is	Estimated Cost of Equipment Eliminated in Reducing Capacity	Estimated Cost of Property Remaining	Depreciation Due to Change In Use Per Cent
1	$ 729,902	$ 92,382	$ 637,520	12.66
2	797,105	55,792	741,313	7.00
3	684,699	191,471	493,228	27.96
4	778,194	47,497	730,697	6.10
5	896,068	26,333	869,735	2.94
6	642,053	34,513	607,540	5.38
7	720,716	176,383	544,333	24.47
8	757,907	118,787	639,120	15.67
	$6,006,644	$743,158	$5,263,486	12.37

The depreciation due to change in use may be computed from the formula

$$\frac{(C - C_a + \sum_{i=0}^{N} pw[O_i - (O_a)_i])}{C} \times 100$$

C = Original Investment or Book Cost
C_a = Equivalent Cost of Adequate Equipment
O_i = Operating Expenses Excluding Depreciation per Fiscal Period
$(O_a)_i$ = Operating Expenses Excluding Depreciation per Fiscal Period Associated with Adequate Equipment

Now, in this example O and O_a are equal, thus depreciation due to decline in use, is expressed by

$$\frac{C - C_a}{C} \times 100 =$$

$$\frac{6{,}006{,}644 - 5{,}263{,}486}{6{,}006{,}644} \times 100 = 12.37\%$$

Other Causes of Depreciation

There are other causes of depreciation besides deterioration, obsoleteness and decline in use. These include the effects of market changes which result in inadequacy; the actions of public authorities, such as highway commissions, public service commissions, police departments, etc., which require the abandonment, modification, shifting, derating or renovating of property, which is otherwise functioning in a satisfactory manner.

The measurement of these effects rests entirely on subjective tests, excepting in those instances where differential costs may be used as they were, in the determination of obsoleteness.

Combination of the Several Elements of Depreciation

The several causes of depreciation have been treated independently up to this point. There is, however, a difficulty in combining the effects. A wholly deteriorated plant which is unaffected by obsoleteness or any other of the causes of depreciation, is nevertheless valueless. Likewise, any property which is subject to the full effects of any other single cause of depreciation is valueless, regardless of the magnitude of the other effects. The problem becomes complicated when the property has experienced partial deterioration, partial obsoleteness and partial decline in use.

The problem may be studied by assuming a property which has experienced no depreciation, and which is made up of 25 value units.

The depreciation due to each cause is zero when the property is new, and this situation may be represented as follows:

Depreciation as Caused by		Corresponding Condition
Deterioration	= 0%	100%
Obsoleteness	= 0%	100%
Decline in Use	= 0%	100%
Other Causes*	= 0%	100%
Loss in Units	= 0%	Remaining Units = 25

* Such as Inadequacy or Requirements of Public Authority.

Some time after installation, an examination of the property reveals that there has been a 20% decline in use, but no change in the physical condition or other aspects which might have caused additional depreciation.

The 20% change in use may have affected all of the property in such a way that any particle will display a 20% change in use, or it may be that all of the change is concentrated in some part of the property, represented, let us say, by 5 value units. These are the limits between which variations may occur. Restating these limits:

Alternative 1. Each value unit may have suffered a change in use of 20%.
Alternative 2. 5 value units may be useless and 20 value units may have suffered no decline in use.

In tabular form, the situation may be visualized as follows:

Alternative 1

Value Unit	Deterioration	Obsoleteness	Decline in Use	Other
1	0%	0%	20%	0%
2	0	0	20	0
3	0	0	20	0
4	0	0	20	0
5	0	0	20	0
6	0	0	20	0
25	0	0	20	0
—	—	—	—	—
Total 25	0%	0%	20%	0%

Alternative 2

Value Unit	Deterioration	Obsoleteness	Decline in Use	Other
1	0%	0%	100%	0%
2	0	0	100	0
3	0	0	100	0
4	0	0	100	0
5	0	0	100	0
6	0	0	0	0
25	0	0	0	0
Total 25	0%	0%	20%	0%

The situation for either Alternative No. 1 or No. 2 now is:

Depreciation as Caused by		Corresponding Condition
A Deterioration	= 0%	100%
B Obsoleteness	= 0%	100%
C Decline in Use	= 20%	80%
D Other Causes	= 0%	100%
Loss in Units	= 5	Remaining Units = 20

Some time elapses before the property is re-examined, at which time some obsoleteness has occurred, although other elements of depreciation have remained unchanged. The obsoleteness that has set in represents a loss in value of 20% and uniformly permeates all of the property, including those portions already subject to change in use. The obsoleteness of 20%, applicable to the 5 value units lost due to decline in use, amounts to 1 value unit. The obsoleteness applicable to the entire property equals 5 value units, so that the obsoleteness applicable to the otherwise undepreciated portions equals 5 — 1 = 4 value units.

It may be found that obsoleteness as well as change in use does not uniformly permeate the property, but is concentrated in different proportions in different portions of the property. The analysis which follows, employs the following designations, *A* as deterioration, *B* as obsoleteness, *C* as change in use, and *D* as depreciation due to other causes, such as inadequacy or requirements of public authority.

The analysis proceeds by examining the four alternatives delimiting the extreme variations.

Alternative 1. Uniform distribution over each particle of property.

Alternative 2. Concentration of obsoleteness in 20% of the property leaving the rest unaffected, where the distribution of change in use is uniform.

Alternative 3. Concentration of change in use in 20% of the property leaving the rest unaffected, where distribution of obsoleteness is uniform.

Alternative 4. Concentration of both deterioration and obsoleteness in the same 20% of the property.

Value Unit	Alternative 1				Alternative 2			
	A	B	C	D	A	B	C	D
1	0%	20%	20%	0%	0%	100%	20%	0%
2	0%	20%	20%	0%	0%	100%	20%	0%
3	0%	20%	20%	0%	0%	100%	20%	0%
4	0%	20%	20%	0%	0%	100%	20%	0%
5	0%	20%	20%	0%	0%	100%	20%	0%
6	0%	20%	20%	0%	0%	0%	20%	0%
25	0%	20%	20%	0%	0%	0%	20%	0%
25	0%	20%	20%	0%	0%	20%	20%	0%

Value Unit	Alternative 3				Alternative 4			
	A	B	C	D	A	B	C	D
1	0%	20%	100%	0%	0%	100%	100%	0%
2	0%	20%	100%	0%	0%	100%	100%	0%
3	0%	20%	100%	0%	0%	100%	100%	0%
4	0%	20%	100%	0%	0%	100%	100%	0%
5	0%	20%	100%	0%	0%	100%	100%	0%
6	0%	20%	0%	0%	0%	0%	0%	0%
25	0%	20%	0%	0%	0%	0%	0%	0%
25	0%	20%	20%	0%	0%	20%	20%	0%

Alternative 1

Loss in Value due to *B* or *C*	= 20%
Remaining Value	= 100% — 20% = 80%
Effect of *C* or *B* respectively	= 20% Additional Loss
	= 20% of 80% + 20%
	= 16% + 4%
Total Loss in Value	= 20% + 16% = 36%
Overlapping Loss	= 4%

Alternative 2

Loss of Value due to *B*	= 20%
Remaining Value	= 100% — 20% = 80%
Effect of *C*	= 20% Additional Loss
	= 20% of 80% + 20%
	= 16% + 4%
Total Loss in Value	= 20% + 16% = 36%
Overlapping Loss	= 4%

Alternative 3

Loss in Value due to *C*	$= 20\%$
Remaining Value	$= 100\% - 20\% = 80\%$
Effect of *B*	$= 20\%$ Additional Loss
	$= 20\%$ of $80\% + 20\%$
	$= 16\% + 4\%$
Total Loss in Value	$= 20\% + 16\% = 36\%$
Overlapping Loss	$= 4\%$

Alternative 4

Loss in Value due to *B* or *C*	$= 20\%$
Remaining Value	$= 100\% - 20\% = 80\%$
Effect of *C* or *B* respectively	$= 20\%$ Additional Loss
But the Overlap	$= 20\%$
Total Loss in Value	$= 20\%$

The demonstration leads to the conclusion that if three factors or more are distributed uniformly throughout each value unit, then the loss or depreciation is 36 per cent of the total value. This result would be obtained by multiplying the several condition factors. The term *condition* is the complement of *depreciation* on a base of 100 per cent.

The following procedure confirms this conclusion, for note should be taken of the fact that in Alternative 4 only two factors permeate all elements, and two do not. The result of multiplying the condition factors results in an indicated loss of 36 per cent, whereas it has been shown that the actual loss is only 20 per cent.

Alternative 1. $(100 - 0) \times (100 - 20) \times (100 - 20) \times (100 - 0) = 64\%$
Depreciation $= 100 - 64 = 36\%$

Alternative 2. $(100 - 0) \times (100 - 20) \times (100 - 20) \times (100 - 0) = 64\%$
Depreciation $= 100 - 64 = 36\%$

Alternative 3. $(100 - 0) \times (100 - 20) \times (100 - 20) \times (100 - 0) = 64\%$
Depreciation $= 100 - 64 = 36\%$

Alternative 4. $(100 - 0) \times (100 - 20) \times (100 - 20) \times (100 - 0) = 64\%$
Depreciation $= 100 - 64 = 36\%$

We can demonstrate algebraically that if a series of condition factors are determined for a property, and all of them are uniformly applicable to each particle of property, the single overall condition factor applicable to the property as a whole may be determined by multiplying together all of the condition factors. Likewise, if in a series of condition factors all *but one* are uniformly applicable to each particle of property, but that *one* is concentrated and not uniformly distributed, then the single

overall condition factor may, nevertheless, be obtained by multiplication of all of the condition factors. However, if more than one of a series of factors is concentrated or lacks uniformity of distribution then this simple solution cannot be applied.

Let A, B, C, D be titles of several types of condition factors.

Let a, b, c be condition factors that are uniformly distributed.

Let x and y be condition factors that are not uniformly distributed.

Value Unit	A	B	C	D	Overall
1	a	b	c	x_1	$abcx_1$
2	a	b	c	x_2	$abcx_2$
3	a	b	c	x_3	$abcx_3$
–	–		–	–	–
–	–		–		–
	–		–	–	–
–	–		–	–	–
n	a	b	c	x_n	$abcx_n$
Total Overall Condition $\div n$	$\frac{\Sigma a}{n}$	$\frac{\Sigma b}{n}$	$\frac{\Sigma c}{n}$	$\frac{\Sigma x}{n}$	$\frac{abc\Sigma x}{n} = abc\frac{\Sigma x}{n}$
Average Overall Condition	a	b	c	$\bar{x}$	$abc\bar{x} = abc\bar{x}$

The following demonstration is intended to show that if more than one factor has not permeated, then the result of multiplying the several condition factors is erroneous.

Value Unit	A	B	C	D	Overall
1	a	b	y_1	x_1	aby_1x_1
2	a	b	y_2	x_2	aby_2x_2
3	a	b	y_3	x_3	aby_3x_3
–		–	–	–	–
	–	–	–	–	–
–	–	–	–	–	–
–	–	–	–	–	–
n	a	b	y_n	x_n	aby_nx_n
Total Overall Condition $\div n$	$\frac{\Sigma a}{n}$	$\frac{\Sigma b}{n}$	$\frac{\Sigma y}{n}$	$\frac{x}{n}$	$\frac{ab\Sigma yx}{n} = ab\frac{\Sigma yx}{n}$
Average Overall Condition	a	b	$\bar{y}$	$\bar{x}$	$ab\bar{y}\cdot\bar{x} \neq ab\frac{\Sigma yx}{n}$

APPENDIX

The bending moment resistance of a pole may be computed as follows:

$$S = \frac{Mc}{I} \text{ or } SI = Mc \quad \text{or} \quad \frac{SI}{c} = M$$

S = The allowable fibre stress in pounds of force per square inch

S for certain common woods

5600	lbs.	per	sq.	in.	western red cedar
3600	"	"	"	"	northern white cedar
3600	"	"	"	"	cypress
6000	"	"	"	"	chestnut
7400	"	"	"	"	creosoted southern yellow pine

M = the resistance moment in inch pounds, or *strength*
c = the distance from the center of the pole to the outermost fibre in inches at the ground line
I = moment of inertia in inches^4

also $I = \frac{\pi d^4}{64}$

d = ground line diameter of pole in inches
d^1 = original diameter
d^2 = diameter of sound wood after rot has taken place

$$M = \frac{\frac{S\,\pi d^4}{64}}{c}$$

$c = r = \frac{1}{2}d$ where r = radius at ground line in inches

$$M = \frac{\frac{S\,\pi d^4}{64}}{\frac{1}{2}d} = \frac{S\,\pi d^3}{32}$$

Poles which are identical except for a ground line diameter have bending moments which stand in the same ratio as the diameters cubed. Because the value of M varies as d^3 as rot proceeds, d is reduced and consequently M is reduced in proportion to the ratio of $\frac{d_2^{\,3}}{d_1^{\,3}}$

PART B—(see page 58)

R_n = Revenue or income per fiscal period
r = Rate of return on capital remaining in business

C = Original investment
d_i = Rate of depreciation which may be a variable
O_n = Operating expenses, excluding depreciation, per fiscal period
n = Number of fiscal periods over which depreciation d_1C has been accrued

From this it can readily be seen that the gross income less all operating expenses excluding depreciation is:

$$R_n = r(C - \sum^{N} d_iC) + d_nC + O_n$$

$$R_n - O_n = r(C - \sum_{i=0}^{N} d_iC) + d_nC$$

$$V = \sum_{i=0}^{N} pw(R_n - O_n) = \sum_{i=0}^{N} pw\left[(r)(C - \sum_{i=0}^{N} d_iC) + d_nC\right]$$

V = Value

The situation which develops following progress of the art influences operating expenses and necessary investment. A new and substitute plant available could produce the same amount of equivalent product as the existing plant, excepting that the investment and operating expenses of the substitute would be different from those of the existing one. The equation applicable to the substitute would assume the general form:

$$(R_e)_n - (O_e)_n = r(C_e - \sum_{i=0}^{n} d_iC_e) + d_nC_e$$

$$V_e = \sum_{i=0}^{N} pw\left[(R_e)_i - (O_e)_i\right] = \sum_{i=0}^{N} pw\left[r(C_e - \sum_{i=0}^{N} d_iC_e) + d_nC_e\right]$$

It is essential to recognize that in the competitive market there is a supply-price curve, which means that at any given price there is an associated quantity of product supplied. Furthermore, at any given price and concomitant quantity of product there is a given total revenue. Now, if the revenues or incomes of both the substitute and the existing plant are the same, then

$$\sum_{i=0}^{N} (R_e)_i = \sum_{i=0}^{N} R_i$$

and

$$\sum_{i=0}^{N} R_i = \sum_{i=0}^{N} pw\left[r(C_e - \sum_{i=0}^{N} d_iC_e) + d_iC_e + (O_e)_i\right] =$$

$$\sum_{i=0}^{N} pw\left[r(C - \sum_{i=0}^{N} d_iC) + d_iC + O_i\right]$$

also $\sum_{i=0}^{N} R_i = V_e + \sum_{i=0}^{N} pw(O_e)_i = V + \sum_{i=0}^{N} pwO$

$$V = V_e - \left[\sum_{i=0}^{N} pwO_i - \sum_{i=0}^{N} pw(O_e)_i \right]$$

but $V_e = C_e$ (i.e., the value of the substitute plant is equal to its cost)

$$V = C_e - \left[\sum_{i=0}^{N} pwO_i - \sum_{i=0}^{N} pw(O_e)_i \right]$$

If it is assumed that the original investment in the existing plant is C and that its present value in the light of progress in the art is V, then loss in value is $C - V$.

$$100 \frac{(C - V)}{C} = \text{Per cent loss in value due to obsoleteness}$$

$$= \frac{C - C_e + \sum_{i=0}^{N} pwO_i - \sum_{i=0}^{N} pw(O_e)_i}{C} 100$$

$$= \frac{C - C_e + \sum_{i=0}^{N} pw\,[O_i - (O_e)_i]}{C} 100$$

C, the original investment, may be stated in terms of original cost as shown on the book of accounts; in terms of historical cost where the books of accounts are incomplete or erroneous and where studies have been made to price each unit of property at the cost which prevailed at the time of purchase or installation; or in terms of cost of reproduction new. Regardless of how C may be stated the investment in the modern substitute plant C_e must be on the same basis. The purpose of this is to eliminate from consideration any differences between C and C_e which may be due to differences between the price levels. Obsoleteness as a cause of depreciation is not brought about by changes in the price level. Convenience dictates that C and C_e be stated in terms of reproduction cost but this is not essential. This study is not intended to explore the relation of price level changes to value. It is contended that a fall in price level, if it should diminish value, is *not* depreciation.

The validity of these equations rests on the assumption that revenues are the same for the existing plant and for the substitute and that the periods over which the present worths are considered are also the same. The savings in operating expenses may be offset by the increased investment and so there may be no obsoleteness. Also, there may be obsoleteness, if there is a decreased investment, even without savings in operating expenses.

CHAPTER VII

ANNUAL DEPRECIATION ALLOWANCE CONSISTENT WITH ESTIMATED ACTUAL DEPRECIATION

Annual depreciation is the loss in value which has occurred between the beginning and the end of the year. The annual allowance for depreciation may be an approximation of this annual loss. It is the annual allowance which is charged to operating expense and credited to the depreciation reserve. The aim should be to make the allowance coincide as nearly as possible with the annual loss. This process may require that an estimate be made at the beginning of each year, so that the charges may be absorbed in small increments throughout that year. If at the end of each year it were practicable to re-estimate the actual depreciation, there would be no problem in ascertaining annual depreciation, for the difference between two succeeding estimates plus the net retirements in that interval yields the required answer. But the estimation of the accrued depreciation is too difficult and costly to be undertaken so frequently, and recourse must, therefore, be had to a short-cut method. Re-estimation of actual accrued depreciation should be made every five or ten years for checking and adjustment purposes.

For convenience in analyzing the effects of depreciation, the several elements thereof may be treated in two categories; one includes those elements which appear to cause more or less uniform depreciation with respect to time and which are apparently subject only to chance variation; the other includes those elements which appear to cause depreciation following no predictable function of time, or which cause depreciation haphazardly.

Short time interval forecasts may be made for a property composed of a large number of units, the use of which has been about the same over a long period of time, on the assumption that the rate of wear and tear or deterioration may be estimated by reference to past experience. In other words, the average rate of deterioration which obtained in the past may be assumed to apply for a short increment of time in the future. This rate is, however, uncorrected for factors which are known to influence its magnitude. Maintenance is such a factor, because it tends to restore some of the property which has been dissipated through wear and tear, and hence the uncorrected rate is greater than it should be by an amount representing the offsetting effect of maintenance.

While wear and tear or deterioration may be assumed to proceed uniformly with time for short increment forecasts, the other elements of depreciation may not be so treated. As a first consideration, the occurrences of these elements follow no predictable function of time, and as a second consideration, there is no certainty that they will ultimately lead to the retirement of property. The development of the steam turbine, for example, has resulted in making the steam engine obsolete to some extent, yet the latter is still being built and it is not now known if or when it will become completely obsolete. Unchecked deterioration will, however, certainly lead to retirement.

Provision for elements in the second category may be made by introducing an allowance for contingency in each annual allowance for depreciation. Periodic review of the status of the reserves and of the actual depreciation of the property, should serve as a guide as to the adequacy or reasonableness of such contingency allowances. In fact, these may be given some foundation by a recognition of the influences of progress of the art or of decline in use, as soon as they occur.

Influences of this type being unpredictable, no definite or reasonable forecast can be made. The most that can be hoped for is prompt recognition of pertinent facts as these transpire, that is, come to be known.

A method which determines a rate of depreciation allowing only for deterioration or wear and tear follows. There are certain facts essential to such a method, and where these are uncertain or unpredictable, it is necessary to make certain assumptions which, of course, influence the accuracy of the final result.

1. It is necessary to know the length of time the dollars devoted to fixed capital have been so applied, and that such dollars have not been devoted to fixed capital via maintenance charges necessary to replace property. It is assumed that each year's additions have been made uniformly throughout that year; furthermore, it is assumed that each year the retirements have been made uniformly throughout that year; furthermore, it is assumed that the dollars retired each year are of an average age at the time of retirement, equal to the weighted average age of the fixed capital dollars out of which the retirements were made.

   $B_n =$ Fixed Capital Balance at Time of Study
   $Y_n =$ Age of B_n
   $R_n =$ Retirements from B_n

2. The accumulated number of dollars of maintenance which have been expended on the property still extant may be determined from the op-

erating expense records, assuming that such expenditures were spread evenly over the physical property, so that for every retirement of fixed capital dollars a proportionate number of maintenance dollars were also retired.

b_n = Maintenance Balance
a_n = Maintenance Expenditure in Year n
r_n = Retirements from b_n in Year n

3. The determination of age of the dollars in the maintenance balance is based on the following assumptions:
   a. Additions to maintenance balance (annual maintenance expenditures) are made uniformly throughout the year.
   b. Retirements from maintenance balance were made uniformly throughout the year.
   c. Dollars of retirement are of an *age* equal to the weighted average *age* of the maintenance balance at the time of retirement.

y_n = Age of the Maintenance Balance

4. The ratio of effectiveness of dollars of value restored by maintenance expenditures, to the dollars of maintenance expenditures is based on assumptions made by engineers where actual studies are impracticable.

I = Ratio of Effectiveness

The maintenance of property usually involves the expenditure of money, which may be analyzed as a composite of two elements. One element paves the way for the successful application of the other. For example, the replacement in kind of the roof of a house may restore that house to its original value. The replacement probably costs more than the original roof, which was placed as a part of the entire original construction, with all tools, facilities and workmen present on the site. The maintenance replacement probably requires the erection of scaffolding, the haulage of tools and materials, and would probably cause some damage to the interior of the house and other special types of charges not originally incurred. The excess of cost is the element which paved the way for the application of the actual maintenance operation. If the restoration of capital is the result sought, and if the total expenditure is greater than this amount, then the ratio of the amount restored to the total maintenance expenditure may be viewed as a *ratio of effectiveness.*

5. The relative rates of the deterioration, or wear and tear, of the original property and of that portion restored through maintenance are based

on the assumption that the original property and that portion which has been restored through maintenance, have deteriorated at the same average rates. In other words, if X_1 represents the rate of deterioration of the original property, and X_2 represents the rate of deterioration of the property which was restored through maintenance, then it is assumed that $X_1 = X_2$ or $X_1 \div X_2 = 1$. For purposes of our subsequent analysis, let us use the symbol X for both rates of deterioration. It is necessary to solve for X in order to determine the average past rate of deterioration.

The determination of the physical condition of the property, C_n, is based on observation and on all facts which may form guides to judgment. The property, at the time of such determination, is a composite of some original property and some replaced property. The dollars representing the present composite property are, similarly, a composite of remaining original dollars and remaining effective maintenance dollars. Thus the "observed" condition may be presented as an equation of the following form:

$$C_n = 1 - Y_n X + \frac{b_n}{B_n} \cdot I(1 - y_n X)$$

C_n, the physical condition of the property, has been determined as previously described. It is not practical to redetermine it at the end of each year and it is therefore desirable to predict the condition as of the end of each year, subsequent to the original determination. An actual determination should be made periodically, so that the predictions may be adjusted and verified from time to time.

$C_n =$ Determined condition
$C_{n+1} =$ Predicted condition as of the end of the next year
$C_{n+2} =$ Predicted condition as of the end of the second year

$$C_n = 1 - Y_n X + \frac{b_n}{B_n} \cdot I(1 - y_n X) \qquad X = \frac{1 - C_n + \frac{b_n}{B_n}(I)}{Y_n + \frac{b_n}{B_n}(I)y_n}$$

$$C_{n+1} = 1 - Y_{n+1} X + \frac{b_{n+1}}{B_{n+1}} I(1 - y_{n+1} X)$$

$$C_{n+2} = 1 - Y_{n+2} X + \frac{b_{n+2}}{B_{n+2}} I(1 - y_{n+2} X)$$

Furthermore

$$B_{n+1} = B_n + A_n - R_n \quad \text{and} \quad b_{n+1} = b_n + a_n - r_n$$

where

$$r_n = b_n \cdot \frac{R_n}{B_n}$$

$$Y_{n+1} = \frac{(B_n - R_n)(Y_{n+1}) + \frac{A_n}{2}}{B_{n+1}}$$

$$y_{n+1} = \frac{(b_n - r_n)(y_{n+1}) + \frac{a_n}{2}}{b_{n+1}}$$

These equations state mathematically that the average age of the balance at the end of the year is equal to the product of the dollars present at the beginning of the year, less the retirements during the year, times the average age at the beginning of the year plus 1, to which is to be added one-half of the additions during the year, all divided by the balance at the end of the year.

A_n, R_n, and a_n are determined by management policy and may be based on budget estimates or trends of past experience.

Considering only deterioration or wear and tear, the reserve provisions may be forecast for any year subsequent to the determination of C_n.

Beginning of Year	*Fixed Capital Balance*	*Remaining Dollars of Value*	*Beginning of Year: Depreciation Reserve*
n	B_n	$C_n(B_n)$	$B_n(1 - C_n)$
$n + 1$	B_{n+1}	$(C_{n+1})(B_{n+1})$	$(B_{n+1})(1 - C_{n+1})$

In normal accounting practice, the annual retirement R_n is debited to the reserve, $B_n(1 - C_n)$. Thus, after the retirement is taken out of the reserve, there is a remainder equal to $B_n(1 - C_n) - R_n$, and by the beginning of the following year this remainder must be made equal to $(B_n + 1)(1 - C_n + 1)$. During the year, therefore, it is necessary to credit the reserve with an amount equal to

$$[(B_{n+1})(1 - C_{n+1})] - [B_n(1 - C_n) - R_n]$$

This would be the amount required for annual depreciation were it not for certain other factors, among which are costs of removal and salvage, which influence R_n.

It should be recalled that in an earlier discussion reference was made to the necessity for adjusting annual depreciation for the offsetting effect of salvage. This is the point in the method where such adjustment is made.

Let us substitute the *net retirement loss,* $R_{n(net)}$ for R_n where

$$R_{n(net)} = R_n + \text{Cost of Removal} - \text{Salvage}$$

then the annual credit to the reserve becomes

$$[(B_{n+1})(1 - C_{n+1})] - [B_n(1 - C_n) - R_{n(net)}]$$

The value of C_{n+1}, computed from the above, may be used as an estimate of the actual physical condition as of one year hence.

So far, consideration has been given only to the physical condition of property as affected by wear and tear, or deterioration. If M_n is a multiplier representing an estimate of all elements of depreciation in the second category, in which all of the haphazard elements fall, then the required number of dollars (D_n) in the depreciation reserve is

$$D_n = B_n(1 - [C_n]\,[M_n])$$

C_n corresponds to the overall physical condition of property due to deterioration, and M_n is the product of all of the other condition factors representing obsoleteness, change in use, public authority, etc.

It has already been stated that only physical condition may be assumed to decline uniformly with time. At this stage of our knowledge, it is not possible to formulate a function to yield $M_n + 1$ in a way similar to that in which $C_n + 1$ was determined. The only alternative is to allow for changes in physical condition during the year and to allow for the effects of all other causes of depreciation by a contingency allowance.

The reserve at the end of the year is:

$$D_{n+1} = B_{n+1}(1 - [C_{n+1}]\,[M_n])$$

The annual allowance for depreciation neglecting contingencies is given by $(D_n + 1) - (D_n)$, the difference between the balances at the beginning and end of the year.

$$D_{n+1} - D_n = [B_{n+1}\,(1 - [C_{n+1}]\,[M_n])] \\ - [B_n(1 - [C_n]\,[M_n]) + R_{n\,(net)}]$$

If the allowance for contingencies is K, then the factor $(1 + K)$ applied to the entire expression will inject this consideration into the determination.

Example Illustrating Determination of Annual Depreciation

The following example is intended to illustrate the application of the method of estimating annual depreciation, using actual data taken from the books of accounts relative to one type of property, owned by a certain public utility company. The accompanying table contains recapitulations of the balances at the beginning of each year for a particular capital account, as well as the related additions and maintenance expenditures made during each year. The other columns are all computed, using these basic data and the computations are discussed step by step in the succeeding pages.

Year (1)	Fixed Capital Balance at Beginning of Year B_n (2)	Additions to Fixed Capital During Year A_n (3)	Retirements from Fixed Capital During Year R_n (4)	Weighted Average Age of Fixed Capital at Beginning of Year Y_n (5)	Maintenance Expenditures During Year a_n (6)	Ratio of Fixed Capital Retirements to Fixed Capital Balance $R_n \div B_n$ (7)	Portion of Maintenance Dollars Applied to Fixed Capital Remaining $a_n - r_n - b_n$ (8)	Portion of Maintenance Balance Removed with F.C. Retirements $r_n - (7) \times (8)$ (9)	Weighted Average Age of Maintenance Balance at Beginning of Year y_n (10)
	$	$	$		$		$	$	
1900	0	15,792	39	0.00					
1901	15,753	9,923	104	0.50					
1902	25,572	528	50	1.11					
1903	26,050	3,361	48	2.08					
1904	29,363	2,769	0	2.78					
1905	32,132	4,065	0	3.50			No record available		
1906	36,198	4,553	1,549	4.05					
1907	39,201	18,162	1,158	4.51					
1908	56,204	6,058	0	3.91					
1909	62,262	6,881	0	4.47					
1910	69,143	11,254	1,234	4.98					
1911	79,162	21,241	5,703	5.20					
1912	94,700	9,769	4,734	4.92					
1913	99,734	24,162	18,290	5.39	2,220	0.1834	0	0	0.00
1914	105,606	10,159	7,823	5.04	1,465	0.0741	2,220	165	0.50
1915	107,943	6,061	707	5.52	2,715	0.0065	3,520	23	1.08
1916	113,297	22,897	49	6.20	2,134	0.0004	6,212	2	1.39
1917	136,145	8,020	1,200	6.07	1,792	0.0088	8,344	73	1.91
1918	142,965	6,790	485	6.70	1,814	0.0034	10,063	34	2.48
1919	149,270	43,656	50	7.37	1,938	0.0003	11,843	4	3.02
1920	192,875	41,057	0	6.59	1,930	0.0000	13,777	0	3.52
1921	233,933	171,037	902	6.35	1,840	0.0039	15,307	60	4.12
1922	404,068	225,712	0	4.45	2,561	0.0000	17,087	0	4.62
1923	629,780	331,946	141,794	3.68	3,052	0.2252	19,648	4,425	4.95
1924	819,932	337,956	0	2.99	1,772	0.0000	18,275	0	5.04
1925	1,157,887	122,010	17,827	2.97	2,555	0.0154	20,047	309	5.55
1926	1,262,071	158,101	25,271	3.64	5,312	0.0200	22,293	446	5.86
1927	1,394,900	94,068	122,607	4.17	4,918	0.0879	27,159	2,387	5.62
1928	1,366,361	383,337	17,394	4.85	9,492	0.0127	29,690	377	5.61
1929	1,732,903	450,839	53,218	4.66	3,639	0.0307	38,805	1,191	5.12
1930	2,130,524	38,294	694	4.57	2,747	0.0000	41,253	0	5.62
1931	2,168,124	296,491	3,797	5.48	5,219	0.0018	44,000	79	6.24
1932	2,460,818	40,209	29,650	5.76	3,080	0.0120	49,140	590	6.52
1933	2,471,377	325,657	77,648	6.66	1,237	0.0314	51,630	1,621	7.10
1934	2,719,386	20,590	7,205	6.80	7,582	0.0026	51,246	133	7.92
1935	2,732,771	13,618	9,185	7.75	5,087	0.0034	58,695	200	7.83
1936	2,737,204	26,264	42,564	8.71	7,269	0.0156	63,382	992	8.16
1937	2,720,904			9.62			69,859		8.26

CONCLUSION

The depreciation policies of corporations have been used in strange ways in the past. In times of unusual prosperity, the belief that regulatory commissions would use the magnitude of possible net earnings as an excuse for trying to lower rates has caused managements to make unusually large retirements or unusually large allowances for depreciation. In times of stress, the allowance has often been reduced to insignificant proportions.* In cases before the New York State Commission, it has been brought out in testimony that equipment long since withdrawn from service, due to various causes, has, nevertheless, been kept on the books.[25]

The recognition that some consistent depreciation policy must be inaugurated, is attested to in the growing demands of public authorities.[26, 27, 28] There is an undercurrent of opinion that if a regulatory body could enforce "age-life" depreciation policy, a long step in the right direction will have been taken.[29]

* Wilson, Herring, Eutsler, *Public Utility Regulation,* McGraw-Hill, 1938.

Though commercial control of depreciation practices is now generally accepted, some utility representatives take an extreme view and contend that the amount of the depreciation charge should be left entirely to the discretion of the utility management. The claim is made that the management should be left free to vary the amount of the annual depreciation charges in order that the amount of the depreciation allowance may be made to conform to the company's financial needs, reducing it in times of stress and increasing it when earnings are large.

Also see testimony of S. Z. Mitchell in 100 Docket No. 14700 Depreciation Charges Telephone Companies and Docket No. 15100 Depreciation Charges Steam R. R.[30a]

Walter Beidatch, *The Functioning of the Massachusetts System of Gas and Electric Utility Regulation,* University of Wisconsin Ph.D. Thesis, page 302.

It seems evident that depreciation is not viewed by the companies as the periodic retrieval from current incomes of the value which has at some particular time been embodied in the fixed assets of the concerns. Rather, the general financial condition of the gas and electric industry, the strategy of rate cases, and dividend policies are the chief determinants of depreciation allowances.[30b]

Joint Legislative Committee to Investigate Public Utilities, State of New York Final Report, page 61.

Letter from the General Superintendent of Transportation of the Consolidated Gas Co., written to the Vice-President in 1933 stated that there were 28 electric vehicles and one Ranier which were obsolete and "could only be disposed of at junk prices." The book value of this obsolete equipment was $102,069.86. The Vice-President replied that "We are of the mind that under present conditions the above stated investment should remain undisturbed, and that, if it becomes necessary, you can defend on the witness stand its inclusion as a part of the rate base on the ground that the equipment is retained for emergency service, when, as and if needed."[32]

The report of the Special Committee on Depreciation of the National Association of Railroad and Utilities Commissioners, in 1938, stated that in respect of straight line or sinking fund depreciation,

> No matter how carefully estimates of service life and salvage are made, actual results may not conform exactly to the estimates *** of course, theoretically, changes in estimates of service lives require adjustments of the depreciation reserves ***

In other words, these advocates of straight-line depreciation recognize the necessity for taking account of facts when and as they occur, for the purpose of modifying estimates of lives determined from past experience.

The adoption of the method of estimating actual depreciation, which would take into account the facts when and as they occur, without the added complication injected by the use of actuarial studies is the more direct procedure. It is recognized that the techniques suggested here are but a beginning and that full development must wait upon use and testing thereof.

Any life-age method requires that voluminous, detailed and costly records be kept of innumerable units of property. The installation of some life-age method of depreciation accounting may have been one of the reasons which urged a number of state commissions to require utilities to install and maintain continuing property record systems. It is probably true that the uniform systems of accounts prescribed by the Federal Power Commission would be unmanageable so far as a number of the requirements are concerned without the installation and maintenance of continuing property record systems although the specific requirement therefore has not been made a part of the regulations or rules. These will furnish accurate information, not only as to original costs, but also as to the age distribution of the property surviving at any time. The record has a further purpose, in that it will make possible the rapid and accurate recapitulation of inventory items.[33]

It is, of course, desirable to have stability in rates. The fluctuations of operating costs, which include allowances for depreciation, determined by estimates of the types described here, might be a serious objection to the method. But, as has been pointed out before, annual allowances for depreciation follow a uniform pattern between times of redetermination of actual accrued depreciation. Examinations of price trends of other important cost factors, such as fuel, are not smooth, and it has been recognized that rates should not be adjusted following each varia-

tion in costs. Stability in rates is brought about by the universal resistance to changes regardless of earnings, except after several years of experience under changed conditions.

In passing, it may be worth noting that a plan has been advanced recently,[30] which, if adopted, would help to standardize depreciation policies, to stabilize expenses so far as allowances for depreciation are concerned, and to spread the risk of rapid changes in the art, which are unpredictable and have caused so much havoc in the past. Dr. Heymann's provocative plan for insurance against depreciation would make available some of the funds necessary at time of retirement, and would urge such retirement upon engineering and economic considerations alone. This method of providing for depreciation, because it is spread on a broad base, might effectively utilize actuarial methods. Dr. Heymann states the case as follows:

> In the personal-life insurance field, the basis of risk has been well established from experience which spreads over generations and many millions of lives. This experience is constantly checked and perfected. *** So in the case of capital risks, and of a system of insurance against them, it is necessary to examine depreciation* and obsolescence experience with buildings, machinery and equipment over a range of years and types. *** Industry has been forced to carry such risks in the form of self-insurance through bookkeeping expedients.

Until such time as the utility industry adopts an insurance plan of some type, the estimated actual depreciation method would appear to yield the safest, most equitable and simplest solution.

This method keeps the owners' funds in the business as a rate base, for a reasonable period of time, and yet provides for the return of these funds, under most conditions, through annual depreciation allowances. To be sure the safest method would be the one described as "installation expense" depreciation, but this would return the funds almost as soon as invested and would require the owners to be on a constant search for new places to invest. The antithesis is the retirement expense method, which keeps the funds in the rate-base for the longest possible time, with a grave risk that situations will occur, requiring the retirement of large blocks of property without appropriate offsetting reserves.

If rates of return are more or less stable, and if there is some reasonable assurance that the investment will be returned, then it would ap-

* Dr. Heymann does not use the generic term *depreciation* to designate loss in value due to all causes.

pear troublesome and costly to require owners to seek new places to re-invest their funds at short periods of time or at frequent intervals. The retirement expense method yields no security but maximizes continuity; the straight line and sinking fund methods greatly increase the security, especially if the actual lives have been underestimated, but the continuity is reduced. In other words, building up reserves to a greater extent than is consistent with reasonable safety, diminishes the rate base, thereby forcing the owners to speed up the search for new places in which to re-invest the reimbursements of capital.

The estimated actual depreciation method yields a reasonable reserve because of its very nature, and yields a return on a larger rate base than either sinking fund or straight line, although this rate base is smaller than in the case of retirement expense accounting.

A test conducted with the accounts of one company resulted in the following:

Fixed Capital After Approximately 47 Years of Operations	$221,000,000
Reserves Required by Straight Line Method	78,800,000
" " " Retirement Expense Method	0
" " " Sinking Fund Method	54,600,000
" " " Estimated Actual Depreciation Method	36,300,000
Annual Depreciation on Basis of Straight Line	9,600,000
" " " " " Retirement Expense	3,500,000
" " " " " Sinking Fund	7,900,000
" " " " " Estimated Actual Depreciation	5,600,000
Fixed Capital Depreciated on Basis of Straight Line	142,200,000
" " " " " " Retirement Expense	221,000,000
" " " " " " Sinking Fund	166,400,000
" " " " " " Estimated Actual	184,700,000
7% Return on Depreciated F.C. Based on Straight Line	9,954,000
" " " " " " " Retirement Expense	15,470,000
" " " " " " " Sinking Fund	11,648,000
" " " " " " " Estimated Actual	12,929,000

Earlier in this discussion reference was made to a theoretical property having an infinite life; here we are dealing with an actual example having property of finite life.

It is hoped that in dealing with this troublesome problem, some paths of further inquiry have been opened, which others may explore. Perhaps we may ultimately have in our possession the knowledge with which to make objective tests of the effects of the several causes of depreciation. It is too early in our experience to shut the door to such exploration by forcing the adoption of age-life calculations, not that

these may not ultimately be found to yield the best results, but that these have not at this time been proved. Before plunging into assumptions as to the future based on the experience of the past, it would be well to tread cautiously. A philosopher has said:[81]

> When we deal with indefinitely large groups and formulate our correlations on the basis of a number of typical or representative instances, no amount of care to see that our samples are taken widely and at random will guarantee us against the fallacy of selection, i.e. against the possibility of our selected instances having some property which makes us pick them out but which is not characteristic of the group as a whole.

A first reading of these conclusions might give the impression that meticulous continuing property records and original cost finding have no other purpose than that of making possible age-life depreciation calculations. This would be false; such efforts make possible the retirements of property at appropriate original costs. As stated in 1938 annual report of the N.Y.P.S.C. at page 11:

> Accounting is the handmaiden of regulation. Proper methods of keeping books and records are the mirrors in which the entire field of utility operations are reflected in detail. . . . All of the new systems (of accounts) require the companies to record the original cost of the properties and establish continuing property records.

The industry may well ponder over the trend of events, which seem to lead directly to the compulsory adoption of accrued and annual depreciation based on age-life methods. This is much to be preferred to the intransigent adherence to methods that have proved disastrous or unfair in the past. This does not mean that a half-hearted attempt at estimating actual depreciation is desirable. In fact such efforts will discredit the whole process of making estimates of actuality. In the face of such testimony as caused the New York Commission to reject "observed depreciation" (see N.Y.P.S.C., 1938, Vol. 1, pp. 511–548) one must sympathize with the regulatory bodies which are aiming at the adoption of straight line methods (see also Rochester Gas & Electric Corp., Case 9733 before N.Y.P.S.C., and "A Decade of Regulation," 1940 Report N.Y.P.S.C.).

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